Rui Wang, Yi-xuan Zhou, Yan-liang Jin, Wen-ming Cao. Sparse fast Clifford Fourier transform. *Frontiers of Information Technology & Electronic Engineering*, 2017, **18**(8):1131-1141. https://doi.org/10.1631/FITEE.1500452

Sparse fast Clifford Fourier transform

Key words: Sparse fast Fourier transform (SFFT); Clifford Fourier transform (CFT); Sparse fast Clifford Fourier transform (SFCFT); Clifford algebra

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Motivation

- The Clifford Fourier transform (CFT) can be applied to both vector and scalar fields. However, it is not efficient in dealing with big data problems because the algorithm is calculated in each semaphore.
- The sparse fast Fourier transform (sFFT) theory deals with the big data problem by using input data selectively.
- The concept of sFFT inspired us to combine CFT with sFFT, which can greatly improve the computing performance in scalar and vector fields.

Main idea

- A new Fourier transform algorithm called sparse fast CFT (SFCFT) is proposed which combines sFFT with CFT to operate on multivector signals.
- The combination can reduce useless or remote data, thus the Fourier transform of multivector signals can be computed more efficiently.

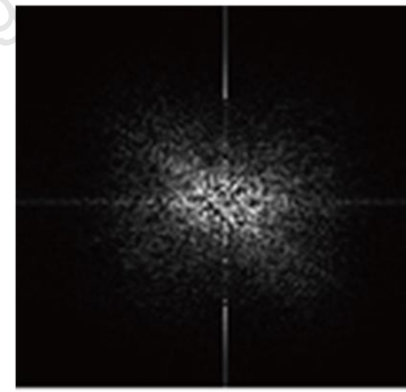
Method

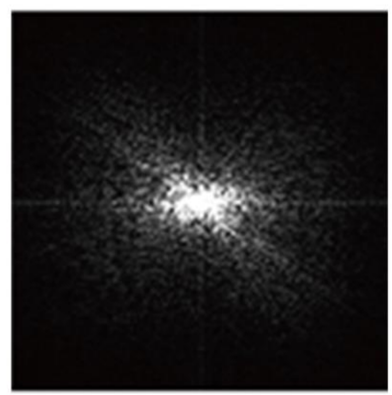
1. We use Clifford algebra to divide a multivector into a Clifford basis {1, e_1, e_2, ...} and transform a Clifford basis into complex signals, then we calculate the Fourier transform of each complex signal separately.
2. The primary algorithm works by first ‘locating’ a set of loops that contain most of the Fourier coefficient.
3. We calculate the *k*-sparse Fourier transform, and with the property of linearity, we add up each transform to obtain the final transform.

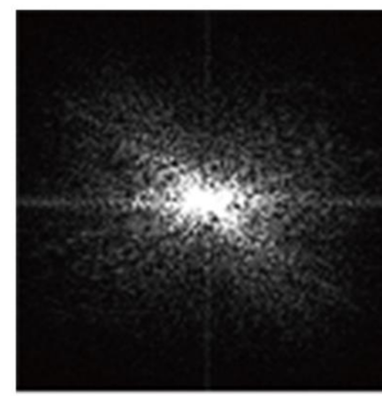
Major results(1)
Performance using 2D grayscale images

- The SFCFT algorithm has the shortest runtime and outputs a clearer spectrum map.
- The original image (a) and the frequency spectrum maps with FFT (b), CFT (c), and SFCFT (d) (The runtimes of FFT, CFT, and SFCFT are 0.029599 s. 0.054489 s, and 0.006536 s, respectively).


(a)

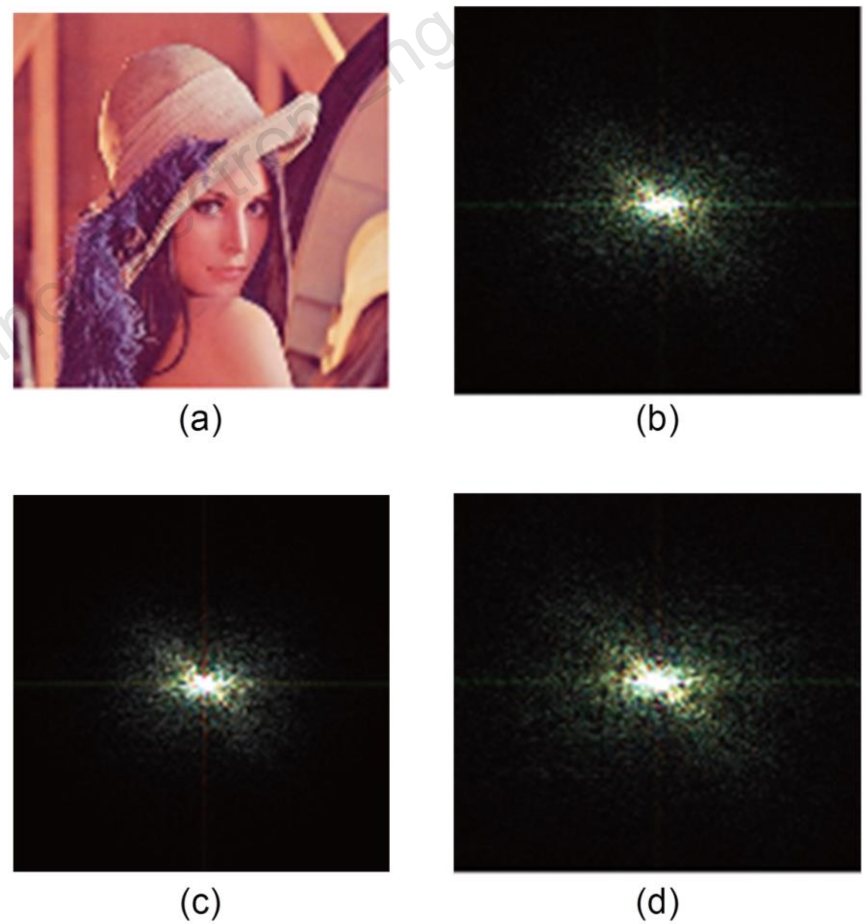

(b)


(c)


(d)

Major results(2)
Performance using color images

- The SFCFT algorithm is much faster than FFT and CFT, and its frequency spectrum map is clearer.
- The original RGB image (a) and the frequency spectrum maps with FFT (b), CFT (c), and SFCFT (d) (The runtimes of FFT, CFT, and SFCFT are 0.195382 s. 0.774559 s, and 0.016852 s, respectively).



Conclusions

- SFCFT transforms the original field into a field where nonzero Fourier coefficients exist. This improves SFCFT's robustness to noise.
- SFCFT can discover both the geometric and spectral information of the multispectral image. The wider the bands of the multispectral image, the more outstanding the SFCFT.
- The experiment results demonstrate that SFCFT can effectively improve the performance of multivector signals processing.