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Research Article

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New formula for predicting plastic buckling pressure of steel torispherical heads under internal pressure

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Abstract: Torispherical heads are commonly used as end closures of pressure vessels. Thin-walled torispherical heads under internal pressure can fail by plastic buckling because of compressive circumferential stresses in the head knuckle. However, existing formulas still have limitations, such as complicated expressions and low accuracy, in determining buckling pressure. In this paper, we propose a new formula for calculating the buckling pressure of torispherical heads based on elastic-plastic analysis and experimental results. First, a finite element method based on the arc-length method to calculate plastic buckling pressure of torispherical heads is established, considering the effects of material strain hardening and geometrical nonlinearity. The buckling pressure results calculated by the finite element method in this paper have good consistency with those of BOSOR5, which is a program for calculating the elastic-plastic bifurcation buckling pressure based on the finite difference energy method. Second, the effects of geometric parameters, material parameters and restraint form of head edge on buckling pressure are investigated. Third, a new formula for calculating plastic buckling pressure is developed by fitting the curve of finite element results and introducing a reduction factor determined from experimental data. Finally, based on the experimental results, we compare the predictions of the new formula with existing formulas. It is shown that the new formula has a higher accuracy than existing ones.

Key words: Torispherical head; Plastic buckling; Elastic-plastic analysis; Prediction formula; Finite element method

1 Introduction

Torispherical heads are commonly used as end closures of pressure vessels in a variety of fields, including the petrochemical, aerospace, and food processing industries. Figure 1 shows the geometry of a torispherical head made up of a spherical crown, a toroidal knuckle, and a short cylinder. These heads are subjected to internal pressure in many actual engineering situations. Due to compressive circumferential stresses in the head knuckle, local buckling can occur in internally pressurized torispherical heads, especially thin-walled ones. As a result, avoiding the failure of buckling has become an important issue in their design.

The buckling of torispherical heads under internal pressure is a classic problem. In 1959, Galletly (1959) pointed out the existence of compressive circumferential stresses in their knuckles , which may lead to buckling failure. Fino and Schneider (1961) reported a case of bucking of a large torispherical head with a diameter of about 18 m in 1961. Bushnell (1976, 1977a) developed the BOSOR4 and BOSOR5 programs for bifurcation buckling analysis. BOSOR4 is used to analyse the buckling of elastic structures with large deformation. BOSOR5 can solve problems involving large deflections, elastic-plastic material behavior and creep, and it can be used for the analysis of plastic buckling and creep buckling. Aylward, Galletly, Bushnell et al. did a series of studies on the buckling of torispherical heads under internal pressure with the use of the programs BOSOR4 and BOSOR5 (Galletly, 1981, 1986a, 1986b; Bushnell, 1977b; Bushnell and Galletly, 1977; Aylward and Galletly, 1979; Galletly and Radhamohan, 1979).

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They studied the buckling behavior of torispherical heads and developed formulas for predicting their buckling pressure. Recently, Li et al. (2017, 2019a, 2019b) and Zheng et al. (2018, 2020a, 2020b) conducted a series of failure experiments and FE analysis on the failure of ellipsoidal and torispherical heads. Based on the experimental results and FE analysis, they have developed new formulas for the prediction of buckling pressure and collapse pressure of ellipsoidal heads under internal pressure. Recent research on the new theory and design of ellipsoidal heads is found in the book *New Theory and Design of Ellipsoidal Heads for Pressure Vessels* (Zheng and Li, 2021). Although ellipsoidal heads can be geometrically equivalent to torispherical heads in some pressure vessel codes, research has shown that ellipsoidal heads have higher buckling resistance compared to equivalent torispherical heads (Zheng, et al., 2020a). Błachut (2020) studied the impact of local and global shape imperfection on the buckling of externally pressurized domes. The results showed that the local dimple created by a concentrated force on the point of maximum deflection leads to a significant reduction of buckling pressure. Błachut (2023) studied the effect of auxetic material on the buckling of externally pressurized torispherical heads and found that the inclusion of auxetic material can lead to 89% gain in failure pressure compared with an auxetic-free configuration of the wall. Zhu et al. (2022) studied the buckling characteristics of spherical shells damaged by concentrated impact load using an experiment and numerical calculation. They found that the buckling load of spherical shells decreases with the increase of impact velocity and impact angle. Yang et al. (2021) deduced the thickness distribution of the constant strength of ellipsoidal heads under external pressure. The FE analysis showed that ellipsoidal heads with variable thicknesses had a higher buckling load than ellipsoidal heads with constant thickness. Zhang et al. (2022) studied the buckling of externally pressurized torispherical heads with uniform and stepwise thickness and found that thickening the whole knuckle and partial crown of the head was the best approach to strengthening them. Sowiński (2020, 2023a, 2023b) focused on stress distribution optimization of dished heads under internal pressure and developed a unique shape to minimize the von Mises stress.

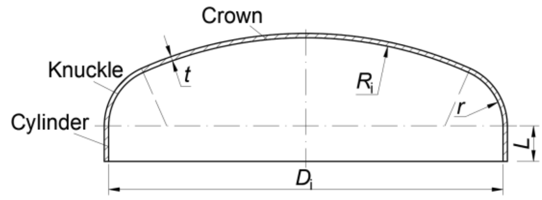


Fig. 1 Geometry of a torispherical head (D_i is inside diameter, L is cylinder length, r is knuckle radius, R_i is crown radius, t is head thickness)

The current formulas for predicting the buckling pressure of torispherical heads under internal pressure are summarized here. Using the program BOSOR4, Aylward and Galletly (1979) developed a formula for elastic buckling pressure of torispherical heads, as given by

$$P_b = 167E\alpha_1\alpha_2\alpha_3\left(t / D_i\right)^{\beta} \tag{1}$$

where, E is elastic modulus, P_b is buckling pressure, $\alpha_1 = 1.1(R_i/D_i)^2 - 1.5R_i/D_i + 1.0$, $\alpha_2 = 48.0(r/D_i)^2 - 6.0r/D_i + 1.0$, $\alpha_3 = [-35.0(r/D_i)^2 + 8.0r/D_i - 0.32](R_i/D_i)^2 + 1$, $\beta = 2.05 + 0.4R_i/D_i$.

The ranges of applicability of Formula (1) are $500 \le D_i/t \le 2000$, $0.75 \le R_i/D_i \le 1.50$ and $0.06 \le r/D_i \le 0.15$. For torispherical heads with $R_i/D_i = 1.0$, Formula (1) can be reduced to:

$$P_b = 100\left(3.7r / D_i + 0.68\right)\left(t / D_i\right)^{2.45} \tag{2}$$

Galletly and Radhamohan (1979) developed a formula for plastic buckling pressure of torispherical heads using an elastic perfectly plastic material model, as given by

$$P_b = \frac{285\left(1 - 125S_y / E\right)\left(r / D_i\right)^{0.84}}{\left(D_i / t\right)^{1.53}\left(R_i / D_i\right)^{1.1}} \tag{3}$$

where, S_y is yield strength. The ranges of applicability of Formula (3) are $500 \le D_i/t \le 1500$, $0.75 \le R_i/D_i \le 1.5$, $0.06 \le r/D_i \le 0.18$ and $138\ \text{MPa} < S_y < 517\ \text{MPa}$. According to the mechanical properties of steels used for most torispherical heads, the average values of yield strength S_y and Young's modulus E are taken as 310 MPa and 200 GPa, respectively. Formula (3) then becomes:

$$P_{\mathrm{b}}=\frac{230S_{\mathrm{y}}\left(r/D_{\mathrm{i}}\right)^{0.84}}{\left(D_{\mathrm{i}}/t\right)^{1.53}\left(R_{\mathrm{i}}/D_{\mathrm{i}}\right)^{1.1}} \tag{4}$$

Galletly and Błachut (1985) expanded the parametric analysis ranges of head geometric parameters and material yield strength. In particular the diameter-thickness ratio D_i/t was expanded down to 300. The prediction formulas were obtained by fitting the buckling pressure results calculated by the BOSOR5 computer program. Two formulas for predicting buckling pressure were developed by Formulas (5) and (6). Formula (6) has a smaller fitting error than Formula (5) but is more complex in form.

$$P_{\mathrm{b}}=\frac{120S_{\mathrm{y}}\left(r/D_{\mathrm{i}}\right)^{0.81}}{\left(D_{\mathrm{i}}/t\right)^{1.46}\left(R_{\mathrm{i}}/D_{\mathrm{i}}\right)^{1.18}} \tag{5}$$

$$P_{\mathrm{b}}=\frac{200S_{\mathrm{y}}\left(r/D_{\mathrm{i}}\right)^{1.5}}{\left(D_{\mathrm{i}}/t\right)^{1.42}\left(R_{\mathrm{i}}/D_{\mathrm{i}}\right)^{1.17}}[1+0.05\left(r/D_{\mathrm{i}}\right)^{-1.315}] \tag{6}$$

In order to predict the buckling pressure of torispherical heads with high strength (517 MPa < S_y < 724 MPa), Galletly (1981) developed a formula for the buckling pressure of torispherical heads with R_i/D_i = 1.0, as given by

$$P_{\mathrm{b}}=\frac{230S_{\mathrm{y}}\left(r/D_{\mathrm{i}}\right)^{0.84}}{\left(D_{\mathrm{i}}/t\right)^{1.53}\left(R_{\mathrm{i}}/D_{\mathrm{i}}\right)^{1.1}} \tag{7}$$

where A=ln(S_y/1000), B=ln(8r/D_i), C=ln(D_i/1000t). The ranges of applicability of Formula (7) are $500 \le D_i/t \le 1250$ and $0.06 \le r/D_i \le 0.2$., and P_b and S_y are in lbf/in^2.

Miller (1999, 2000) developed formulas for predicting the failure pressure of torispherical heads under internal pressure. The formulas corresponding to yielding pressure and elastic buckling pressure are given by thin shell theory and elastic buckling theory. The formulas for buckling pressure are derived by applying reduction factors to the formulas corresponding to yielding pressure and elastic buckling pressure. The reduction factors obtained from test data account for the effects of geometric imperfections, residual stresses and nonlinearity material properties. Miller's formulas are presented below.

(a) Elastic buckling stress S_e

$$S_{\mathrm{e}}=C_1E\left(t/r\right) \tag{8}$$

where

$$C_1=\begin{cases}9.31r/D_{\mathrm{i}}-0.086, r/D_{\mathrm{i}}\le 0.08\\ 0.692r/D_{\mathrm{i}}+0.605, r/D_{\mathrm{i}}>0.08\end{cases}$$

(b) Maximum membrane stress in knuckle S_k (negative value is compression)

$$S_{\mathrm{k}}=C_2\left(0.5\frac{R_{\mathrm{e}}}{r}-1\right)p\frac{R_{\mathrm{e}}}{t} \tag{9}$$

where, p is internal pressure, and

$$C_2=\begin{cases}1.25, r/D_{\mathrm{i}}\le 0.08\\ 1.46-2.6r/D_{\mathrm{i}}, r/D_{\mathrm{i}}>0.08\end{cases}$$

$$R_{\mathrm{e}}=\begin{cases}\left(0.5D_{\mathrm{i}}-r\right)/\left[\cos\left(\beta_{\mathrm{th}}-\phi_{\mathrm{th}}\right)\right]+r, \phi_{\mathrm{th}}<\beta_{\mathrm{th}}\\ 0.5D_{\mathrm{i}}, \phi_{\mathrm{th}}\ge\beta_{\mathrm{th}}\end{cases}$$

$$\beta_{\mathrm{th}}=\arccos\left(\frac{0.5D_{\mathrm{i}}-r}{R_{\mathrm{i}}-r}\right),\ \phi_{\mathrm{th}}=\frac{\sqrt{R_{\mathrm{i}}t}}{r}$$

(c) Elastic buckling pressure P_e and yield pressure P_y

$$P_{\mathrm{e}}=\frac{S_{\mathrm{e}}t}{C_2R_{\mathrm{e}}\left[\left(0.5R_{\mathrm{e}}/r\right)-1\right]} \tag{10}$$

$$P_{\mathrm{y}}=\frac{S_{\mathrm{y}}t}{C_2R_{\mathrm{e}}\left[\left(0.5R_{\mathrm{e}}/r\right)-1\right]} \tag{11}$$

(d) Buckling pressure of torispherical heads assembled from formed segments

$$P_{\mathrm{b}}=\begin{cases}0.6P_{\mathrm{e}},\ P_{\mathrm{e}}/P_{\mathrm{y}}<1.0\\ 0.408P_{\mathrm{y}}+0.192P_{\mathrm{e}},\ 1.0<P_{\mathrm{e}}/P_{\mathrm{y}}<8.29\\ 2.0P_{\mathrm{y}},\ P_{\mathrm{e}}/P_{\mathrm{y}}>8.29\end{cases} \tag{12}$$

(e) Buckling pressure of pressed and spun torispherical heads

$$P_b = \begin{cases} 0.6P_e, & P_e / P_y < 1.0 \\ 0.408P_y + 0.192P_e, & 1.0 < P_e / P_y < 8.29 \\ 2.0P_y, & P_e / P_y > 8.29 \end{cases} \quad (13)$$

Based on the formulas for calculating buckling pressure, ASME VIII-1 (ASME, 2023b), ASME VIII-2 (ASME, 2023c), and EN 13445-3 (CEN, 2021) provide design formulas for preventing the buckling of internal pressurized torispherical heads. The design formulas used in ASME VIII-1 and VIII-2 are based on the formulas of Miller (1999, 2000). The design formulas used in EN 13445-3 are based on the work by Galletly (1986a, 1986b).

As described above, the existing formulas were developed using elastic or perfectly plastic theories, in which material strain hardening is not considered. In addition, the formulas developed by Galletly et al. are not applicable to a sufficient range. Miller's formulas are quite complicated and thus inconvenient to use. The object of this paper is to develop a new formula to predict the buckling pressure of steel torispherical heads using elastic-plastic analysis and experimental results. First, the nonlinear FE method is established to calculate the buckling pressure of torispherical heads, taking into account the effects of stress hardening and geometrical nonlinearity. Second, the effects of geometrical parameters, material parameters and restraint of head edge on the buckling pressure of torispherical heads are investigated. Third, a new formula for calculating plastic buckling pressure is developed by fitting the curve of finite element results and introducing a reduction factor determined from experimental data. Finally, the predictions of the new formula and existing formulas are compared with experimental results.

2 Numerical simulation of buckling of torispherical heads

2.1 FE model

The nonlinear finite element analysis using the arc-length method is used to simulate the buckling of torispherical heads under internal pressure. In this study, we used the ANSYS software to perform the finite element buckling analysis including the effects of material and geometrical nonlinearities. Figure 2 shows the geometrical model of a torispherical head with a cylinder. The model is assumed to have perfect shape. A torispherical head is usually welded to a long cylinder in actual application. For this model, it is assumed that the cylinder and the torispherical head have the same inside diameter and thickness. The cylinder has such a length that the effect of boundary stresses due to displacement constraint of the cylinder ends is negligible for torispherical heads.

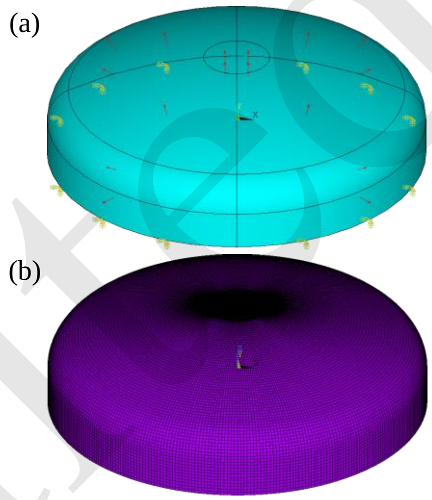


Fig. 2 Boundary condition (a) and mesh (b) of torispherical heads with cylinders

An increasing uniform pressure is applied to the inside surface of the model and the end of the cylinder is fully fixed, as shown in Fig. 2 (a). As buckling mainly occurs in a thin-walled torispherical head with a diameter-thickness ratio greater than 100, the model is meshed by element SHELL181, as shown in Fig. 2 (b). The element SHELL181 is suitable for thin to moderately-thick shell structures and for linear, large deflection, and large strain nonlinear applications.

Most torispherical heads in practice are constructed from steel. The true stress-strain curve including strain hardening characteristics is used for the material model, and von Mises yield criterion and the flow rule are adopted. Table 1 presents tensile properties of some typical steels commonly used for torispherical heads. These tensile properties, including elastic modulus, engineering yield strength and engineering tensile strength are obtained from ASME 2023 BPV code Section II, part D (ASME, 2023a) and Chinese standards GB/T 150.2—2011 (AQSIQ, 2011) and GB/T 24511—2017 (AQSIQ, 2017). The true stress-strain curves of these steels are determined by Annex 3-D of ASME 2023 BPV code Section VIII, division 2 (ASME, 2023c), as shown in Fig. 3. When the curve exceeds the true tensile strength, the material is assumed to be perfectly plastic.

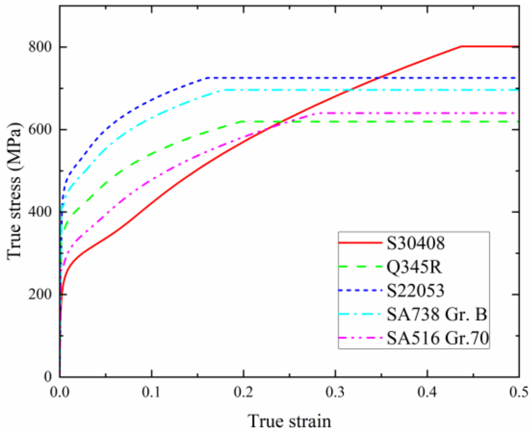


Fig. 3 True stress-strain curves of typical steels

Table 1 Tensile properties of typical steels

Material	Elastic modulus E (GPa)	Engineering yield strength S_y (MPa)	Engineering tensile strength S_u (MPa)
S30408	195	220	520
S22053	200	450	620
Q345R	201	345	510
SA-516 Gr.70	202	260	485
SA-738 Gr.B	202	415	585

2.2 FE results

2.2.1 Determination of buckling pressure

Taking a torispherical head with a crown radius-diameter ratio (R_i/D_i) of 1.0 and a knuckle radius-diameter ratio (r/D_i) of 0.1, such as is commonly used in engineering, as an example. The head and cylinder have the same inside diameter of 5000 mm, the same thicknesses (t) of 5 mm, and the cylinder has a length (L) of 500 mm. The true stress-true strain curve of S30408 is used in the material model. The FE results on buckling of the torispherical head are shown in Fig. 4. It is shown that local buckles occur in the knuckle of the head. The radial displacement of points A, B and C of a buckle was obtained to determine the buckling pressure. The pressure-displacement curves of points A, B and C are nearly identical when the pressure is lower than 0.21 MPa. When it exceeds 0.21 MPa, the radial displacement of point B starts to develop differently from points A and C, which indicates the start of buckling. The buckling pressure of the model is thus determined as 0.21 MPa.

In addition, the pressure-displacement curves are linear when the pressure is lower than 0.12 MPa, meaning that the torispherical head is in elastic deformation. As the pressure increases, plastic deformation occurs and causes the pressure-displacement curves to become nonlinear. Buckles occur subsequently, as shown in Fig. 4. Therefore, the buckling of internally pressurized torispherical heads occurs in the plastic region of the material, as mentioned in Refs. (Galletly and Radhamohan, 1979; Galletly, 1981).

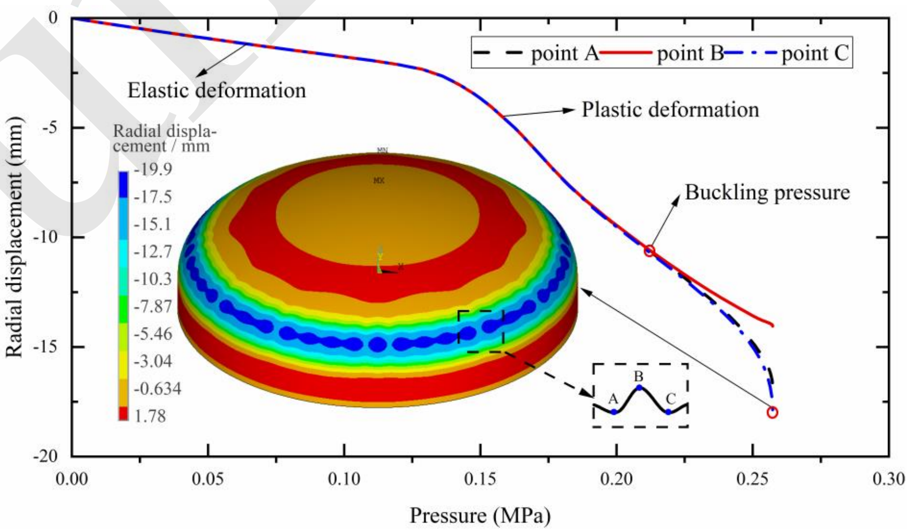


Fig. 4 FE results on buckling of torispherical heads

The FE method in this paper is compared with that in reference (Galletly and Radhamohan, 1979), which uses BOSOR5 to calculate the elastic-plastic bifurcation buckling pressure of torispherical heads. BOSOR5 is a computer program for the buckling of elastic-plastic complex shells of revolution including large deflections and creep (Bushnell, 1976). The pre-buckling and elastic-plastic bifurcation analyses of the program are based on the finite difference energy method. Bifurcation buckling load is computed corresponding to buckling modes. For the initial circumferential wave number of the shell (chosen by the user), BOSOR5 calculates the determinant of the global stability matrix for each load increment. The load is increased until the stability determinant changes sign, which indicates the occurrence of bifurcation buckling. Once the bifurcation load is determined with the initial wave number, BOSOR5 computes the eigenvalues in wave number increments, and the minimum eigenvalue is used to decide on the proper wave number.

The nonlinear finite element analysis in this paper is based on the arc-length method. This method is a highly efficient numerical method in structural nonlinear analysis. It has good adaptability and higher efficiency in the pre- and post-buckling analysis of structures for tracing the whole load-deflection path and capturing the buckling load.

Three torispherical head models were used for comparison, using an elastic perfectly plastic material model with a yield strength of 207MPa and an elastic modulus of 207GPa. The comparison between the calculated buckling pressure in this paper and the reference (Galletly and Radhamohan, 1979) is shown in Table 2. The difference between them is within 5%. It can be seen that the buckling pressure results of torispherical heads calculated by the FE method in this paper have good consistency with those of BOSOR5.

The BOSOR5 and FE methods in this paper are capable of calculating the buckling pressure and the number of circumferential waves of heads at buckling. For example, for a head with D_i/t = 1000, r/D_i = 0.15, and R_i/D_i = 1, both BOSOR5 and the FE method calculate a circumferential wave number of 40 at buckling. However, the FE method can further simulate the post-buckling behavior of the heads, while BOSOR5 cannot.

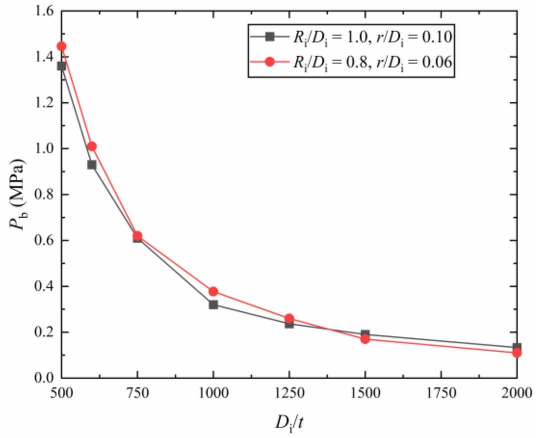
Table 2 Comparision of buckling pressure results calculated by the FE method and BOSOR5

D_i/t	r/D_i	R_i/D_i	Buckling pressure, P_b (MPa)		Difference (%)
			FE Method (This paper)	BOSOR5	
1000	0.15	1	0.255	0.258	-1.2
1500	0.1	1	0.099	0.095	4.2
1000	0.08	1.25	0.122	0.117	4.3

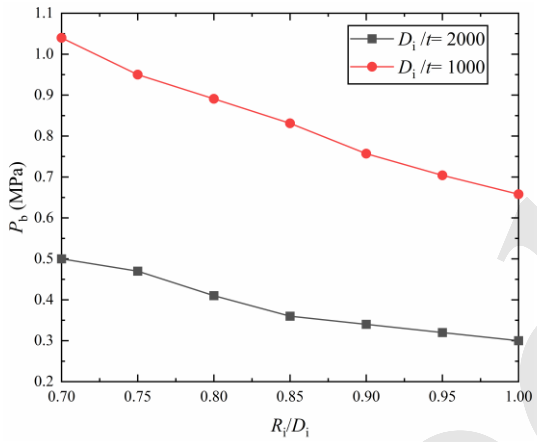
2.2.2 Effects of geometrical parameters

The effects of head diameter-thickness ratio (D_i/t), crown radius-head diameter ratio (R_i/D_i) and knuckle radius-head diameter ratio (r/D_i) on the buckling pressure (P_b) of torispherical heads are studied using the FE model. Figure 5 shows FE results of buckling pressure for some heads with different geometrical parameters. According to the FE results, buckling pressure of heads decreases with the increase of D_i/t and R_i/D_i, and increases with the decrease of r/D_i.

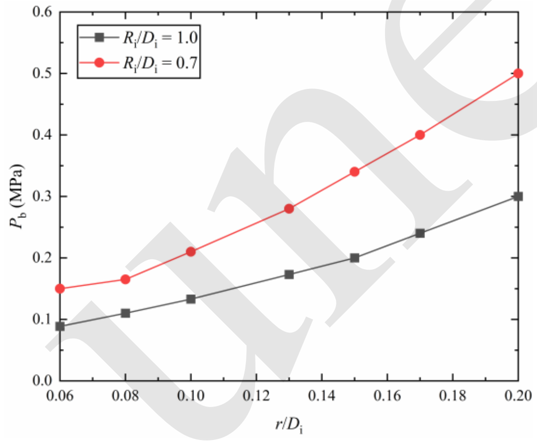
Taking torispherical heads (D_i =5000, D_i/t =2000) with different knuckle and crown radius as an example, the FE method is used to conduct stress analysis of the head. The heads were attached to cylinders with the same diameter and thickness as the head, and the head and cylinder are assumed to be made of S22053. Figure 6 shows the distribution of circumferential stress in the middle surface of the head and cylinder under an internal pressure of 0.09 MPa. As we can see, the circumferential stress in the crown of the head is constant. However, the circumferential stresses fluctuate greatly in the head knuckle because of the curvature discontinuity in the junctions between the crown, knuckle and cylinder. There are significant compressive stresses in the head knuckle, which is the cause of buckling. The maximum compression stresses increase with the increase of R_i/D_i and decrease of r/D_i, which results in a decrease of buckling pressure.



(a) D_i/t



(b) R_i/D_i



(c) r/D_i

Fig. 5 Effects of geometrical parameters D_i/t, R_i/D_i and r/D_i on buckling pressure of torispherical heads

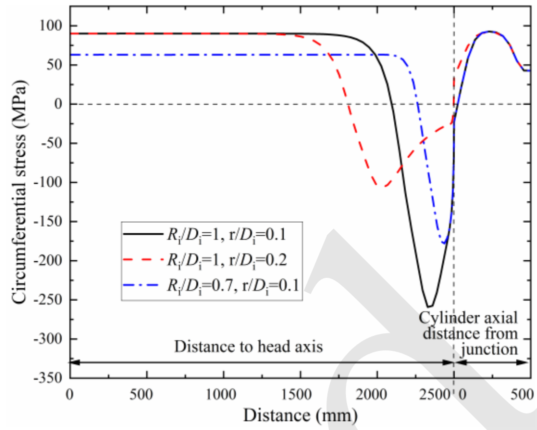


Fig. 6 Circumferential stresses of torispherical heads under internal pressure

2.2.3 Effect of material parameter

In order to study the effect of material parameters on the buckling pressure of torispherical heads, the true stress-strain curves of five kinds of typical steels are used in the FE model, as shown in Fig. 3. The FE results show that buckling occurs in the plastic region of materials for the heads we studied. So the effect of yield strength (S_y) on buckling pressure (P_b) is studied, as shown in Fig. 7. It is shown that buckling pressure increases approximately linearly with the increase of yield strength.

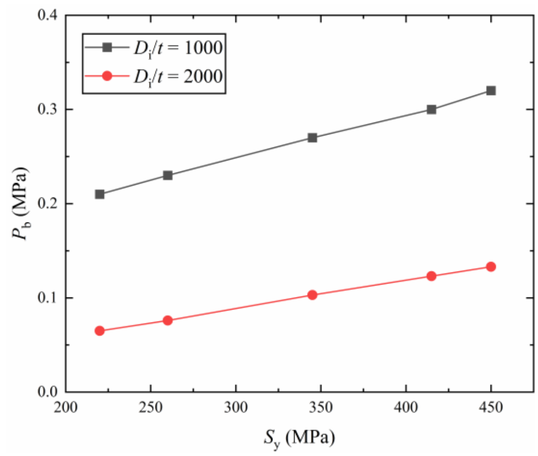


Fig. 7 Effect of yield strength on buckling pressure of torispherical heads under internal pressure

2.2.4 Effect of restraint forms of head edge

Torispherical heads are mainly attached to cylinders, but in some applications, they may also be connected to bolting flanges. When connecting with the cylinder, a cylinder with the same thickness as the head and sufficient length is considered in the FE model. When connecting to the bolting flange, the cylinder is not considered in the FE model, and the edge of the torispherical head is fully fixed. The effects of restraint forms on the buckling pressure of torispherical heads are shown in Table 3. As can be seen, when the head diameter-thickness ratio (D_i/t) is large, the restraint form of head edge has little effect on the buckling pressure. When the head diameter-thickness ratio (D_i/t) decreases, the fixed constraint of head edge will enhance the buckling resistance of torispherical heads.

As shown in Table 3, for a torispherical head with D_i/t = 2000, r/D_i = 0.06, and R_i/D_i = 0.7, the buckling pressure is 0.14 MPa without cylinder, and it decreases to 0.11 MPa when attached to a sufficiently long cylinder with $L=5\sqrt{D_i t/2}$. To further investigate the influence of cylinder length on the buckling pressure, additional calculations were performed for cylinder lengths of $0.5\sqrt{D_i t/2}$, $\sqrt{D_i t/2}$, and $2\sqrt{D_i t/2}$, and the buckling pressures were 0.12 MPa, 0.11 MPa, and 0.11 MPa, respectively. It is evident that the buckling pressure increases as the cylinder length L decreases; however, beyond a certain value of L, the buckling pressure is no longer affected.

Table 3 Influence of restraint forms on the buckling pressure of the torispherical head

D_i/t	Buckling pressure, P_b (MPa)					
	r/D_i =0.06, R_i/D_i = 1		r/D_i =0.20, R_i/D_i = 1		r/D_i =0.06, R_i/D_i = 0.7	
	With cylinder	No cylinder	With cylinder	No cylinder	With cylinder	No cylinder
500	0.91	NB	NB	NB	NB	NB
2000	0.07	0.07	0.22	0.22	0.11	0.14

Note: NB = No buckling

3 Development of formulas for the buckling pressure of torispherical heads

3.1 Formula by curve fitting of FE results

In order to develop a formula for predicting the buckling pressure of torispherical heads under internal pressure, more FE models were generated to carry out a further parametric study. As discussed in Section 2.2.4, such heads attached to long cylinders have lower buckling pressure than those with a fixed edge and the restraint form of the head edge has little effect on buckling pressure when D_i/t is large. Therefore, a long cylinder attached to a torispherical head is considered in the FE models so as to obtain conservative results. The ranges of the geometric parameters in this study are determined as $200 \le D_i/t \le 2000$, $0.7 \le R_i/D_i \le 1.0$ and $0.06 \le r/D_i \le 0.2$, covering the range of applicability of the torispherical heads commonly used in engineering. The five kinds of typical steels are considered in the FE models, and their true stress-true strain curves are shown in Fig. 3.

A total of 149 FE models for torispherical heads with different geometrical parameters and material properties were generated to obtain buckling pressures. The FE results are plotted in Fig. 8. According to the FE results, the buckling pressure has a positive linear correlation with S_y, an exponential decay relationship with D_i/t, R_i/D_i and an exponential growth relationship with r/D_i. Based on these relationships, a formula is obtained by nonlinear curve fitting of these FE results, as shown in Fig. 8. The coefficient of determination value is 0.94, showing a good fitting. Therefore, a new formula for prediction of buckling pressure of perfect torispherical heads under internal pressure is given as

$$P_b = 305 S_y \left(D_i/t\right)^{-1.67} \left(R_i/D_i\right)^{-1.32} \left(r/D_i\right)^{0.47} \quad (14)$$

According to the parameters of the heads used in the FE simulation, Formula (14) is applicable for torispherical heads with $200 \le D_i/t \le 2000$, $0.7 \le R_i/D_i \le 1.0$ and $0.06 \le r/D_i \le 0.2$.

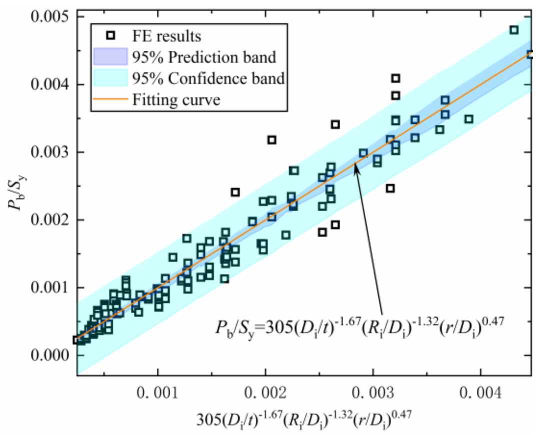


Fig. 8 Curve fitting of FE results for the buckling of torispherical heads

3.2 Formula modified by experimental results

Shape imperfections are inevitable during the manufacturing process of the head, especially for large-scale thin-walled heads. Li K, et al. (2017) calculated the buckling pressure of a large-scale thin-walled head by nonlinear FE analysis considering the initial measured and the initial perfect shapes respectively. The experimental buckling pressure of the head agrees well with that predicted by the analysis considering the initial measured shape. Zheng, et al. (2018) found that shape imperfections have significant effects on the buckling pressure of large-scale thin-walled heads assembled from formed segments. Wagner et al. (2021) validated a wide variety of empirical design and numerical geometric imperfection approaches for torispherical shells under external pressure and developed new design factors for them. Therefore, to develop a new formula for the buckling pressure of actual torispherical heads, Formula (14) should be modified to take into account shape imperfections.

Experimental results on buckling of 49 internally pressurized torispherical heads, including 30 heads formed by spinning and pressing and 19 heads assembled from formed segments, are summarized in Tables 4 and 5. The ranges of geometric parameters of these tested heads are $238 \leq D_i/t \leq 2304$, $0.722 \leq R_i/D_i \leq 1.093$ and $0.04 \leq r/D_i \leq 0.21$. These heads are constructed in carbon steel, low alloy steel and stainless steel. We have conducted a series of internally pressurized buckling experiments for heads (Li et al., 2017, 2019a, 2019b; Zheng et al., 2018; Zheng and Li, 2021). The tested heads were welded to reusable test vessels, and the test vessels were pressurized with water until the heads burst. Four heads formed by cold spinning (ZJU-X1 ~ ZJU-X4) and four heads assembled from formed segments (ZJU-CAP1 ~ ZJU-CAP2, ZJU-STD1 ~ ZJU-STD2) are torispherical heads which are equivalent to ellipsoidal heads. For heads ZJU-X1 ~ ZJU-X4, experimental buckling pressure was determined by the occurrence of the first buckle using video monitoring. For large-scale heads (ZJU-CAP1 ~ ZJU-CAP2, ZJU-STD1 ~ ZJU-STD2), the strain in the knuckle was measured by strain gauges, and the deformation was measured by 3D laser scanners (Li, et al., 2017; Zheng J, et al., 2018). The buckling pressure was determined by the circumferential strain-pressure curves of the first buckle, and the initiation of waves detected by the 3D laser scanners. The experimental results show that buckles form in the knuckles of heads and gradually evolve with increasing pressure. Some typical buckles of tested heads are presented in Fig. 9. For example, for head ZJU-STD2, two buckles were formed at a pressure of 0.72 MPa. Subsequently, as the pressure reached 0.95 MPa, the number of buckles increased to 10 and, finally, 14 buckles occurred on this head (Li K, et al., 2019a). For head ZJU-X3, a total of 13 buckles was finally formed. The rest of the experimental results are obtained from the literature reported by Miller (1999). The experimental buckling pressure of most heads is the pressure at which the first buckle is detected visually. But for some heads such as T11 and T13, it is the pressure at which the first buckle is detected by a pressure drop.

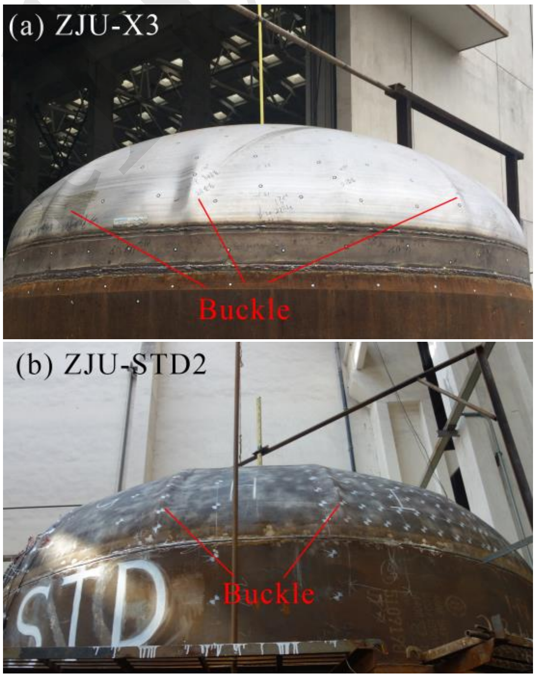


Fig. 9 Photograph of typical buckles in torispherical heads under internal pressure

Table 4 summarizes experimental results for torispherical heads formed by spinning and pressing, and gives comparison of the experimental results and predictions of Formula (14). It can be seen that the relative error between experimental and computational results ranged from −33% to 38%, which is in relatively good agreement.

Table 4 Comparison of predictions of Formula (14) and experimental results for torispherical heads formed by spinning and pressing

Head No.	D_i (mm)	D_i/t	R_i/D_i	r/D_i	Material	Yield strength, S_y (MPa)	Experimental buckling pressure, P_{exp} (MPa)	Formula (14), P_b (MPa)	Relative error (%)
ZJU-X1	1800	450	0.94	0.14	Stainless steel	392	2.51	1.91	−24
ZJU-X4	1200	400	1	0.12	Stainless steel	327	1.45	1.66	14
ZJU-X2	1800	450	1.01	0.12	Stainless steel	392	1.77	1.62	−8
ZJU-X3	1200	400	1	0.12	Stainless steel	340	1.47	1.73	18
KR1	1526.5	238	0.982	0.062	Carbon steel	425	4.14	3.86	−7
T16	500.4	370	1	0.1	Carbon steel	262	1.21	1.39	15
T15	500.4	373	1	0.1	Carbon steel	262	1.3	1.37	5
T10	500.1	355	1	0.04	Carbon steel	270	0.79	1	27
T9	505	368	1	0.04	Carbon steel	239	0.83	0.83	0
T4	502.9	370	1.093	0.06	Carbon steel	280	1.19	1.04	−13
T3	502.9	381	1.093	0.06	Carbon steel	290	1.13	1.03	−9
SC6	1371.6	412	0.778	0.074	Stainless steel	294	1.92	1.58	−18
SC5	1371.6	412	0.833	0.074	Stainless steel	294	1.92	1.44	−25
SC3	1371.6	412	1	0.111	Stainless steel	294	1.71	1.37	−20
SC4	1371.6	412	1	0.074	Stainless steel	294	1.37	1.13	−18
T5	500.1	535	1	0.04	Carbon steel	263	0.39	0.49	26
T6	501.9	543	1	0.04	Carbon steel	279	0.37	0.51	38
T13	500.4	543	1	0.1	Carbon steel	253	0.61	0.71	16
T2	502.9	541	1.093	0.06	Carbon steel	259	0.44	0.51	16
T1	502.9	550	1.093	0.06	Carbon steel	230	0.43	0.44	2
T11	502.9	1068	1	0.1	Carbon steel	253	0.26	0.23	−12
SC17	2057.4	618	0.833	0.074	Stainless steel	294	0.74	0.73	−1
SC11	2743.2	824	0.722	0.074	Stainless steel	294	0.59	0.55	−7
SC16	2057.4	618	1	0.074	Stainless steel	294	0.66	0.58	−12
KN2b	1676.4	629	1	0.076	Stainless steel	207	0.34	0.4	18
SC10	2743.2	824	0.833	0.074	Stainless steel	294	0.54	0.45	−17
SC12	2743.2	824	1	0.056	Stainless steel	294	0.46	0.31	−33
SC9	2743.2	824	1	0.074	Stainless steel	294	0.43	0.36	−16
SC8	2743.2	824	1	0.111	Stainless steel	294	0.48	0.43	−10
K5	4249.4	1044	1	0.083	Stainless steel	294	0.23	0.25	9

Table 5 summarizes experimental results for torispherical heads assembled from formed segments and gives the comparison of buckling pressure between the experimental results and predictions of Formula (14). The relative error between experimental and computational results of Formula (14) ranges from −14% to 57%. The relative error of the heads with small diameters is small, but for large heads with a diameter of 4 - 5 m, the relative error is relatively large, and mostly above 40%. It is because large shape imperfections occur in torispherical heads assembled from formed segments due to welding residual deformation, which causes a decrease of buckling pressure (Li, et al., 2019a). Therefore, a formula for predicting the buckling pressure of torispherical heads assembled from formed segments is obtained by introducing a reduction factor of 0.8 into Formula (14), as given by

$$P_b = 244 S_y \left(D_i / t\right)^{-1.67} \left(R_i / D_i\right)^{-1.32} \left(r / D_i\right)^{0.47} \quad (15)$$

As shown in Table 5, the relative error between experimental and computational results of Formula (15) ranged from −31% to 24%, which is in relatively good agreement.

Table 5 Comparison of predictions of the formulas and experimental results for torispherical heads assembled from formed segments

Head No.	D_i (mm)	D_i/t	R_i/D_i	r/D_i	Yield strength, S_y (MPa)	Experimental buckling pressure, P_{exp} (MPa)	Formula (14)		Formula (15)	
							Calculation results (MPa)	Relative error (%)	Calculation results (MPa)	Relative error (%)
ZJU-CAP1	4797	872	0.78	0.21	509	1.15	1.27	10	1.02	−11
ZJU-CAP2	4797	872	0.78	0.21	588	1.15	1.47	28	1.18	3
ZJU-STD1	5000	909	0.89	0.17	612	0.7	1.08	54	0.86	23
ZJU-STD2	5000	909	0.89	0.17	558	0.7	0.99	41	0.79	13
SC1	1371.6	412	1	0.167	294	1.93	1.66	-14	1.33	−31
SC2	1371.6	412	1	0.167	294	1.92	1.66	-14	1.33	−31
Meesters	2743.2	457	1	0.085	290	0.98	1	2	0.8	−18
CBI2	4876.8	711	0.9	0.17	353	0.73	0.93	27	0.74	1
CBI1	4876.8	980	0.9	0.17	361	0.4	0.56	40	0.45	13
Fino	18288	2304	0.905	0.173	262	0.09	0.1	11	0.08	−11
SC14	2057.4	618	1	0.167	294	0.83	0.84	1	0.67	−19
SC15	2057.4	618	1	0.167	294	0.74	0.84	14	0.67	−9
K4	4249.4	730	0.89	0.162	294	0.48	0.73	52	0.58	21
K1	4249.4	825	0.91	0.159	294	0.37	0.58	57	0.46	24
SC7	2743.2	824	1	0.167	294	0.41	0.52	27	0.42	2
SC13	2743.2	824	1	0.167	294	0.57	0.52	-9	0.42	−26
K2	4249.4	880	1	0.163	294	0.32	0.46	44	0.37	16
K3	4249.4	915	1	0.166	294	0.29	0.44	52	0.35	21
Stennett	1936.8	953	1	0.105	293	0.28	0.33	18	0.26	−7

Combining Formula (14) and Formula (15), a new Formula (16) for calculating buckling pressure of torispherical heads is proposed as follows:

$$P_b = 305\beta_t S_y \left(D_i / t\right)^{-1.67} \left(R_i / D_i\right)^{-1.32} \left(r / D_i\right)^{0.47} \quad (16)$$

where,

$$\beta_t = \begin{cases} 1.0 \text{ for steel torispherical heads fabricated} \\ \quad \text{by spinning and pressing} \\ 0.8 \text{ for steel torispherical heads assembled} \\ \quad \text{from formed segments} \end{cases}$$

Formula (16) is applicable for steel torispherical heads with $200 \le D_i/t \le 2000$, $0.7 \le R_i/D_i \le 1.0$ and $0.06 \le r/D_i \le 0.2$. For the tested heads beyond that scope, the relative error between the calculated buckling pressure of Formula (16) and the experimental results is analysed. For Head No. Fino (D_i/t = 2304), ZJU-X2 (R_i/D_i = 1.01), T1–T4(R_i/D_i = 1.093), ZJU-CAP1 & CAP2 (r/D_i = 0.21), the relative error ranges from −13% to 16%, as shown in Tables 4 and 5. For Head No. T1–T4 (r/D_i = 0.04), SC12 (r/D_i = 0.056), the relative errors range from −33% to 38%, as shown in Table 4. So, for tested heads with $D_i/t >$ 2000, $R_i/D_i > 1.0$, $r/D_i > 0.2$, Formula (16) is still relatively accurate, but for heads with $r/D_i < 0.06$, the calculation error is slightly high.

Table 4, Table 5 and Fig. 10 show the comparison between the new Formula (16) and experimental results. Stripping out heads (r/D_i = 0.04) beyond the range of the formula, the relative error range between the prediction of the new formula and the experimental results is −31% to 24%, and the average absolute relative error is 14%, which shows that the predicted buckling pressures of the proposed new formula for calculating the buckling pressure of the torispherical head agree well with the experimental results.

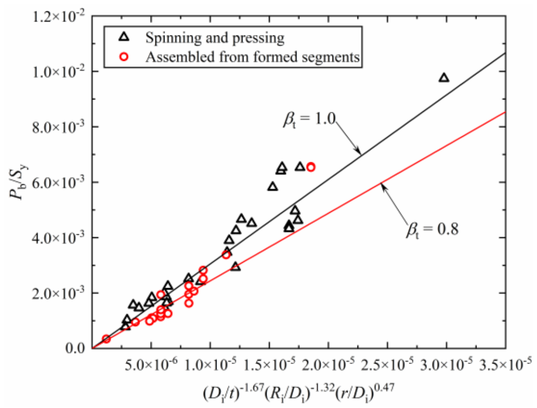


Fig. 10 Comparison of experimental results and Formula (16)

4 Comparison with other formulas

Table 6 lists formulas for predicting buckling pressure of torispherical heads subjected to internal pressure. Aylward & Galletly's formula is used for elastic buckling although the buckling type of commonly used steel heads is usually plastic buckling. Therefore, the calculated buckling pressures of Aylward & Galletly's formula are 2.9–7.1 times the experimental results for steel heads, as shown in Table 7.

Table 6 Summary of formulas for the buckling of torispherical heads under internal pressure

Formula	Formula No.	Limitation	Buckling type
Aylward & Galletly's formula	(1)	$500 \le D_i/t \le 2000$, $0.75 \le R_i/D_i \le 1.5$, $0.06 \le r/D_i \le 0.15$	Elastic buckling
Galletly's formula	(7)	$500 \le D_i/t \le 1250$, $R_i/D_i = 1.0$, $0.05 \le r/D_i \le 0.2$	Plastic buckling
Galletly's formula	(4)	$500 \le D_i/t \le 1500$, $0.75 \le R_i/D_i \le 1.5$, $0.06 \le r/D_i \le 0.18$	Plastic buckling
Galletly & Błachut's formula	(5), (6)	$300 \le D_i/t \le 1500$, $0.8 \le R_i/D_i \le 1.0$, $0.05 \le r/D_i \le 0.2$	Plastic buckling
Miller's formula	(8)–(13)	$20 \le D_i/t \le 2806$, $0.72 \le R_i/D_i \le 1.82$, $0.04 \le r/D_i \le 0.35$	Elastic/Plastic buckling
Proposed in this paper	(16)	$200 \le D_i/t \le 2000$, $0.7 \le R_i/D_i \le 1.0$, $0.06 \le r/D_i \le 0.2$	Plastic buckling

Table 7 Comparison between experimental results and Aylward & Galletly's formula

Head No.	D_i (mm)	D_i/t	R_i/D_i	r/D_i	S_y (MPa)	Buckling pressure (MPa)		$\frac{P_b}{P_{exp}}$
						Experiment, P_{exp}	Formula, P_b	
T13	500.4	543	1	0.1	253	0.61	3.97	6.5
T2	502.9	541	1.093	0.06	259	0.44	3.03	6.9
T1	502.9	550	1.093	0.06	230	0.43	2.91	6.8
T11	502.9	1068	1	0.1	253	0.26	0.76	2.9
SC17	2057.4	618	0.833	0.074	294	0.74	3.23	4.4
SC16	2057.4	618	1	0.074	294	0.66	2.51	3.8
KN2b	1676.4	629	1	0.076	207	0.34	2.42	7.1
SC10	2743.2	824	0.833	0.074	294	0.54	1.63	3
SC9	2743.2	824	1	0.074	294	0.43	1.24	2.9
SC8	2743.2	824	1	0.111	294	0.48	1.48	3.1
K5	4249.4	1044	1	0.083	294	0.23	0.72	3.1
Stennett	1936.8	953	1	0.105	293	0.28	1	3.6

Table 8 presents comparison of the buckling pressure between the existing formulas and experimental results. Galletly et al. developed formulas for plastic buckling pressures of torispherical heads, as described in Section 1. Formula (7) has a limitation in the scope of applicability (it is only applicable to torispherical heads with R_i/D_i = 1.0), thus it is not discussed here.

The relative error range between the prediction of Galletly's Formula (4) and experimental results is −54% to 53%. As for heads formed by spinning and pressing, the predicted buckling pressure of the formula is 6% to 54% lower than the experimental results, and it is 40% to 53% higher for some heads assembled from formed segments. In addition, this formula is not applicable to heads with D_i/t < 500,

$D_i/t > 1500$.

The relative error between the predicted buckling pressure of Galletly & Błachut's Formula (5) and experimental results is −59% to 37%. For heads formed by spinning and pressing, the predicted buckling pressure of the formula is 21% to 59% lower than the experimental results, and the relative error is −37% to 37% for heads assembled from formed segments. In addition, this formula is not applicable to heads with $D_i/t > 1500$.

The relative error between the predicted buckling pressure of Galletly & Błachut's Formula (6) and experimental results is −63% to 34%. For heads formed by spinning and pressing, the predicted buckling pressure of the formula is 22% to 63% lower than the experimental results, and the relative error is −44% to 34% for heads assembled from formed segments. In addition, this formula does not apply to heads with $D_i/t > 1500$.

The relative error between the predicted buckling pressure of Miller's formulas and experimental results is −57% to 23%. The predicted buckling pressure of Miller's formulas is much lower than the experimental results for most tested heads. The reason is that Miller's formulas provide a conservative prediction for experimental results in order to further develop a safe design method for preventing buckling of torispherical heads. But for heads ZJU-X3, ZJU-CAP2, SC7, K1 – K4, the predicted buckling pressure of Miller's formulas is higher than the experimental results.

Table 8 Comparison of different formulas to predict buckling pressure of torispherical heads

Head No.	Experimental buckling pressure, P_{exp} (MPa)	Galletly Formula (4)		Gal&Bla-1 Formula(5)		Gal&Bla-2 Formula(6)		Miller Formulas (8–13)	
		P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)
ZJU-X1	2.51	1.61	−36	1.38	−45	1.25	−50	1.50	−40
ZJU-X4	1.45	1.32	−9	1.12	−23	0.99	−32	1.45	0
ZJU-X2	1.77	1.31	−26	1.12	−37	1	−44	1.25	−29
ZJU-X3	1.47	1.38	−6	1.16	−21	1.03	−30	1.50	2
KR1	4.14	N/A	N/A	N/A	N/A	N/A	N/A	1.88	−55
T16	1.21	1.02	−16	0.87	−28	0.76	−37	1.06	−12
T15	1.3	1.01	−22	0.86	−34	0.75	−42	1.05	−19
T10	0.79	N/A	N/A	N/A	N/A	N/A	N/A	0.46	−42
T9	0.83	N/A	N/A	N/A	N/A	N/A	N/A	0.4	−52
T4	1.19	0.65	−45	0.55	−54	0.51	−57	0.73	−39
T3	1.13	0.64	−43	0.55	−51	0.5	−56	0.73	−35
SC6	1.92	N/A	N/A	N/A	N/A	N/A	N/A	0.85	−56
SC5	1.92	0.93	−52	0.81	−58	0.72	−63	0.83	−57
SC3	1.71	1.07	−37	0.9	−47	0.8	−53	1.15	−33
SC4	1.37	0.76	−45	0.65	−53	0.58	−58	0.79	−42
T5	0.39	N/A	N/A	N/A	N/A	N/A	N/A	0.3	−23
T6	0.37	N/A	N/A	N/A	N/A	N/A	N/A	0.31	−16
T13	0.61	0.55	−10	0.48	−21	0.43	−30	0.61	0
T2	0.44	0.33	−25	0.29	−34	0.27	−39	0.42	−5
T1	0.43	0.29	−33	0.25	−42	0.24	−44	0.36	−16
T11	0.26	0.2	−23	0.18	−31	0.16	−38	0.18	−31
SC17	0.74	0.5	−32	0.45	−39	0.4	−46	0.5	−32
SC11	0.59	N/A	N/A	N/A	N/A	N/A	N/A	0.34	−42
SC16	0.66	0.41	−38	0.36	−45	0.33	−50	0.46	−30
KN2b	0.34	0.29	−15	0.25	−26	0.23	−32	0.32	−6
SC10	0.54	0.32	−41	0.29	−46	0.27	−50	0.3	−44
SC12	0.46	0.21	−54	0.19	−59	0.18	−61	0.22	−52
SC9	0.43	0.26	−40	0.24	−44	0.22	−49	0.27	−37
SC8	0.48	0.37	−23	0.33	−31	0.3	−38	0.33	−31
K5	0.23	0.2	−13	0.18	−22	0.17	−26	0.19	−17
ZJU-CAP1	1.15	N/A	N/A	N/A	N/A	N/A	N/A	1.13	−2
ZJU-CAP2	1.15	N/A	N/A	N/A	N/A	N/A	N/A	1.23	7

Head No.	Experimental buckling pressure, P_{exp} (MPa)	Galletly Formula (4)		Gal&Bla-1 Formula(5)		Gal&Bla-2 Formula(6)		Miller Formulas (8–13)	
		P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)	P_b (MPa)	Relative error (%)
ZJU-STD1	0.7	1.07	53	0.96	37	0.94	34	0.61	−13
ZJU-STD2	0.7	0.98	40	0.88	26	0.85	21	0.57	−19
SC1	1.93	1.5	−22	1.26	−35	1.19	−38	1.72	−11
SC2	1.92	1.5	−22	1.26	−34	1.19	−38	1.72	−10
Meesters	0.98	0.72	−27	0.62	−37	0.55	−44	0.76	−22
CBI2	0.73	0.89	22	0.78	7	0.76	4	0.71	−3
CBI1	0.4	0.56	40	0.5	25	0.49	23	0.4	0
Fino	0.09	N/A	N/A	N/A	N/A	N/A	N/A	0.08	−11
SC14	0.83	0.81	−2	0.7	−16	0.67	−19	0.74	−11
SC15	0.74	0.81	9	0.7	−5	0.67	−9	0.74	0
K4	0.48	0.69	44	0.61	27	0.58	21	0.59	23
K1	0.37	0.55	49	0.49	32	0.47	27	0.44	19
SC7	0.41	0.52	27	0.46	12	0.44	7	0.42	0
SC13	0.57	0.52	−9	0.46	−19	0.44	−23	0.42	−26
K2	0.32	0.46	44	0.41	28	0.39	22	0.36	13
K3	0.29	0.44	52	0.39	34	0.38	31	0.34	17
Stennett	0.28	0.28	0	0.25	−11	0.23	−18	0.22	−21

Figure 11 shows the comparison of prediction accuracy of existing formulas and the new formula proposed in this paper. The new formula has the smallest average absolute relative error of 14%, which is about half that of other formulas. The Root Mean Square Error (RMSE) of the new formula is also the smallest. In general, the prediction accuracy of the new formula is the best among these formulas.

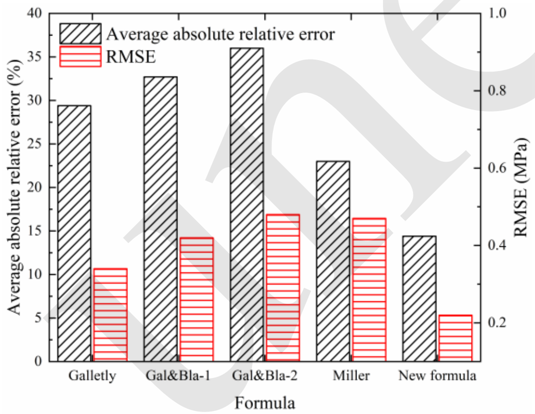


Fig. 11 Comparison of different formulas for buckling pressure of torispherical heads

5 Conclusions

This paper investigates the formulas of plastic buckling pressure for steel torispherical heads under internal pressure. A nonlinear FE method based on the arc-length method is established to calculate the buckling pressure of torispherical heads, taking into account the effects of stress hardening and geometrical nonlinearity. The buckling pressure results calculated by the FE method have good consistency with those of BOSOR5, which is a program for calculating the elastic-plastic bifurcation buckling pressure based on the finite difference energy method. The effects of geometry, material parameters and restraint forms of head edge on buckling pressure were investigated. It shows that buckling pressure of torispherical heads decreases with the increase of D_i/t and R_i/D_i, and increases with the decrease of r/D_i. Buckling pressure increases approximately linearly with the increase of yield strength. The fixed constraint of head edge will enhance the buckling resistance of torispherical heads, but has little effect for heads with large D_i/t.

By fitting the curve of elastic-plastic finite element results and introducing a reduction factor determined from experimental results, a new formula of plastic buckling pressure for steel torispherical heads under internal pressure is proposed. This formula is applicable for steel torispherical heads with $200 \leq D_i/t \leq 2000$, $0.7 \leq R_i/D_i \leq 1.0$, $0.06 \leq r/D_i \leq 0.2$. Compared with Galletly, Galletly & Błachut, and Miller's formulas, the new formula has a comprehensive advantage in terms of its accuracy and applicability.

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Author contributions

Jinyang ZHENG leads the research project. Jinyang ZHENG and Keming LI designed the research. Sheng YE, Keming LI and Shan SUN processed the corresponding data. Sheng YE wrote the first draft of the manuscript. Shan SUN helped to organize the manuscript. Keming LI and Sheng YE revised and edited the final version.

Conflict of interest

Sheng YE, Keming LI, Jinyang ZHENG, and Shan SUN declare that they have no conflict of interest.

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中文概要

题　目： 内压碟形封头屈曲压力计算新公式

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目　的： 屈曲是内压碟形封头的重要失效模式。已有的碟形封头屈曲压力计算公式存在计算精度低、适用范围窄、计算过程繁琐等问题。本文旨在提出具有更高精度和更强适用性的钢制碟形封头屈曲压力计算新公式，为建立内压碟形封头抗屈曲设计方法提供支撑。

创新点： 已有公式基于弹性理论或理想弹塑性理论，未考虑材料应变硬化影响。本文基于考虑几何非线性和材料应变硬化的非线性屈曲有限元分析方法，并经大量工业规模封头屈曲试验数据修正，提出了钢制碟形封头屈曲压力计算新公式，相比已有公式具有更高精度与更强适用性。

方　法： 1.采用基于弧长法的非线性增量有限元方法，建立了考虑材料应变硬化和几何非线性的钢制碟形封头屈曲压力计算模型（图 4）。2. 利用该模型开展钢制碟形封头屈曲压力参数化计算，探明了封头几何参数、材料参数等对其屈曲压力的影响规律（图 5 和 7），结合有限元计算结果和大量工业规模封头屈曲压力试验数据，提出了碟形封头屈曲压力计算新公式（公式(16)）。

结　论： 1. 钢制碟形封头在内压下发生塑性屈曲，屈曲压力随封头径厚比（D_i/t）、球冠区半径与内径比（R_i/D_i）的增大而减小，随过渡区半径与内径比（r/D_i）的减小而增大，与材料屈服强度近似成线性关系；2. 本文提出的钢制碟形封头屈曲压力计算新公式，适用范围为 $200 \le D_i/t \le 2000$, $0.7 \le R_i/D_i \le 1.0$, $0.06 \le r/D_i \le 0.2$，相比现有公式具有更高精度和更强适性。

关键词： 碟形封头；塑性屈曲；弹塑性分析；预测公式；有限元方法