Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering) in press
www.jzus.zju.edu.cn; www.springer.com/journal/11582
E-mail: jzus_a@zju.edu.cn

Research Article
https://doi.org/10.1631/jzus.A2300251

Time-synchronous-averaging spectrum based on super-resolution analysis and application in bearing fault signal identification

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Abstract: Time synchronous averaging (TSA) is based on the idea of denoising by averaging, and it extracts the periodic components of a quasiperiodic signal and keeps the extracted waveform undistorted. This paper studies the mathematical properties of TSA, where three propositions are given to reveal the nature of TSA. This paper also proposes a TSA-spectrum based on super-resolution analysis and it decompose a signal without using any base function. In contrast to discrete Fourier transform spectrum (DFT-spectrum), which is a spectrum in frequency domain, TSA-spectrum is a period-based spectrum, which can present more details of the cross effects between different periodic components of a quasiperiodic signal. Finally, a case study is carried out using bearing fault analysis to illustrate the performance of TSA-spectrum, where the rotation speed fluctuation of the shaft is estimated, which is about 0.12 milliseconds difference. The extracted fault signals are presented and some insights are provided. We believe that this paper can provide new motivation for TSA-spectrum to be widely used in applications involving quasiperiodic signal processing (QSP).

Key words: Time synchronous averaging; Spectrum; Quasiperiodic signal processing; Super-resolution analysis; Bearing fault detection

1 Introduction

This paper presents an innovative approach in the field of quasiperiodic signal processing (QSP) by introducing the Time Synchronous Averaging Spectrum (TSA-spectrum) based on Super-Resolution Analysis. QSP is essential in various applications, from medical science to engineering, where the extraction of periodic components from signals is crucial. While Fourier Transform (FT) has traditionally been used to transform signals from time to frequency domains, we explore the concept of period as an independent parameter in the analysis of quasiperiodic signals. We provide an overview of the significance of QSP, the limitations of traditional frequency-based methods, and the unique characteristics of period-based analysis. In the subsequent section, we delve into the background of our research to offer a comprehensive understanding of the context and motivation behind the proposed TSA-spectrum.

1.1 Background

Using Fourier transform (FT), we can transform a signal from a time domain into a frequency domain. As is known, a period is the reciprocal of a frequency. A spectrum in a frequency domain naturally corresponds to a spectrum in a period domain. More specifically, both the terms ‘period’ and ‘frequency’ mentioned herein are parameters of a sine or cosine function. However, intrinsically, the concept of ‘period’ should be independent from the existence of a

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Received May 9, 2023; Revision accepted Dec. 5, 2023; Crosschecked

sine or cosine function. When we try to liberate 'period' from FT, the liberated period is not the reciprocal of the frequency of any base functions, such as sine, cosine, and wavelet functions. Without a specific base function, the period is a more general concept describing a periodic signal.

Mathematically, a signal $y(\mathrm{t})$ is a periodic signal if there exists a $T \in \mathbb{R}^{+}$ satisfying $\mathrm{y(t)} = \mathrm{y(t+T)}$ for all $t \in \mathbb{R}$. The parameter T denotes the period of $y(\mathrm{t})$.

In the real world, it is rare to observe a perfect periodic signal. Commonly, what we can observe is a signal containing one or more periodic components. We name a nonperiodic signal with hidden periodic components as a quasiperiodic signal, which can be described as

$$\hat{\mathrm{y}}(\mathrm{t}) = \mathrm{y}(\mathrm{t}) + \mathrm{g}(\mathrm{t})$$

where, $\mathrm{y}(\mathrm{t})$ is the periodic component and $\mathrm{g}(\mathrm{t})$ is a nonperiodic term.

In real applications, either $y(t)$ or $\mathrm{g}(\mathrm{t})$ may contain the information of interest. Quasiperiodic signal processing (QSP) refers to signal processing techniques able to identify or extract the information we want, while at the same time eliminating or suppressing other information. The exact purpose of QSP depends on the needs of the actual application.

Quasiperiodic signal processing covers a wide range of application scenarios, from medical science and climate analysis to general usage in engineering. For medical science, QSP can be applied to electrocardiography (ECG) to monitor heart conditions (Lin and Hu, 2008; Birrenkott et al., 2018; Martens et al., 2018; Musuamba et al., 2021;Wang et al., 2023), and QSP techniques can be used with electroencephalograms (EEG) to reveal the activity of the human brain (Thakor and Tong, 2004; Chaumon and Bishop, 2015; Mannan et al., 2018; Mpekris et al., 2020; Nagwanshi et al., 2023.). For applications in engineering, we can apply QSP to analyze the vibration signals of rotating machinery, such as bearings (Randall and Antoni, 2011; Wang et al., 2014; Smith and Randall, 2015; Yao et al., 2022) and gears (Wang et al., 2018; Sun at al., 2018; Touret et al., 2018; Tan et al., 2021), for fault diagnosis and early warning. This can be of crucial importance for the operational safety of high-speed trains (Hong et al., 2014; Chen et al., 2019; Peng et al., 2019; Gabrić et al., 2021), wind turbines (Qiao and Lu, 2015; Salameh et al., 2018; Wang et al., 2019), and engines (Wang et al., 2013; Delvecchio et al., 2018; Ma et al., 2019; Ross, 2023).

Currently, there are many signal processing techniques pertaining to QSP methods, such as FT (Gothwal et al., 2011; Talhaoui et al., 2014; Lee et al., 2014; Sugavanam et al., 2019; Ma & Tao, 2021; Thibault et al., 2023), wavelet transforms (Chen et al., 2016; Wang et al., 2018; Gupta et al., 2019; Tianet al., 2023), and many other filtering methods (Roth et al., 2017; Li et al., 2018; Zhang et al., 2019; Bommert et al., 2020). Almost all these techniques share a common point that relates to a spectral description in the frequency domain. In this paper, we call a signal processing technique a frequency-based method if it analyzes a signal in a frequency domain based on a base function, such as a sine, cosine, or wavelet function. In contrast, we name a technique a period-based method if it analyzes a signal in the period domain. It should be noted that time synchronous averaging (TSA) (McFadden, 1987) can be taken as a typical period-based method for signal processing. TSA is not new, but it is classic and effective. Hereafter, this paper is mainly focused on TSA and we introduce the TSA-spectrum based on Super-Resolution Analysis from the aspect of the period spectrum. A brief review of TSA is presented in Section 1.2.

1.2 Brief review of time synchronous averaging (TSA)

TSA is a technique with a long history and there is much related literature. It is based on the idea of denoising by averaging and it does not rely on any base function. It is widely used in condition evaluation of rotating equipment, such as bearings (Mishra et al., 2016; Yao et al., 2022) and gearboxes (Combet and Gelman, 2007; Halim et al., 2008; Ahamed et al., 2014; Bravo-Imaz et al., 2017; Camerini et al., 2019; Zhang and Hu, 2019; Tan et al., 2021). TSA can extract periodic components from a signal and keep the extracted waveform undistorted. The performance of TSA in processing signals, such as vibration and noise, of a device with rotating structures has been well studied.

TSA is ergodic and its denoise property can be described as follows. Non-synchronous noise is reduced by the reciprocal of the square root of the number of revolutions (McFadden, 1987). The only parameter of TSA is the length of the synchronous signal which, in most publications dealing with bearings or gearboxes, is related to the revolution of the rotating shaft. Usually, a tachometer, or speed sensor, is necessary to provide a reference value of the length of the synchronous signal (McFadden and Toozhy, 2000; Mishra et al., 2016; Schmidt et al., 2021). However, since it is hard to determine accurately the rate of revolution, researchers have proposed solutions to estimate its possible value by some data-driven approaches (Fong et al., 2019; Syed et al., 2022; Zhao et al., 2022).

There are many improvements and applications of TSA, which combine TSA with other signal processing techniques such as wavelet transform (WT) or EMD, including ISTA (Rahman et al., 2011), TSMA (Zhang and Hu, 2019), MIR-TSA (Ahamed et al., 2014), TSA with windows (McFadden, 1991; Smidt, 2010; Pitarresi et al., 2020; Gao et al., 2022), TSA algorithm in frequency domain (McFadden and Toozhy, 2000; Mishra et al., 2016; Roy et al., 2016; Sim et al., 2022). It should be noted that the performance of TSA can be enhanced by resampling before averaging, mostly by interpolation (McFadden, 1989), which is taken as super-resolution analysis of TSA.

1.3 Contribution of this paper

In this paper, we propose the TSA-spectrum for visualizing possible periodic components of a signal in the period domain. While TSA is concise and simple, the mathematical properties concerning performance on quasiperiodic signals are intricate. The essential mathematical characteristics of TSA are presented in three propositions, backed by the congruence theory in number theory. Our work in this paper introduces several significant advances on the existing literature:

(1) Theoretical Framework & Enhanced Visualization: We establish the theoretical framework of TSA with vital definitions and propositions. The TSA-spectrum is introduced to shed light on the periodic components concealed within a signal. In terms of visualization, our method offers an intuitive representation that promotes quicker and more precise fault diagnosis. Furthermore, super-resolution analysis is brought in to augment the efficacy of the TSA-spectrum (Discussed in Appendix A).

(2) Comparison with DFT: The relationship and divergence between the TSA-spectrum and the Discrete Fourier Transform spectrum (DFT-spectrum) are meticulously explored. In a theoretical context, TSA emerges as a pivotal complement to DFT, especially in the long-period (low-frequency) domain. This distinction highlights the precision of the TSA method in capturing and representing cyclic variations, which sets it apart from conventional techniques. (Elaborated in Section 2).

(3) Empirical Validation with Bearing-fault Analysis: We undertake a comprehensive case study focusing on bearing-fault analysis. By deploying the TSA-spectrum on a public dataset of bearing vibration readings, we effectively extract fault signals associated with distinct bearing defects. Such insights are novel, with no similar revelations recorded in the bearing-fault diagnosis sphere. Importantly, our results underline the commendable performance of the TSA-spectrum on QSP, reinforcing its adaptability and versatility in identifying a range of bearing-faults. This, combined with its computational efficiency, paves the way for its potential in real-time applications. (Detailed in Section 3).

(4) In conclusion, our efforts in this paper not only introduce a novel approach in the form of the TSA-spectrum but also validate its strengths and advantages over existing methods in the literature. We trust that these contributions address the current gaps and foster further research and exploration in the domain.

2 The relationship and difference between TSA-spectrum and DFT-spectrum

This section discusses the relationship and difference between the Discrete Fourier Transform (DFT) and TSA. DFT is mainly based on the concept that a finite-energy signal can be described as a combination of a series of sine or cosine functions. In contrast, TSA is derived from the essential characteristics of a periodic signal and is quite different from signal processing techniques based on signal decomposition using sine or cosine functions. A comparison between TSA and DFT is illustrated in Fig. 1.

Taking the rectangular function $S(t; T_0)$ with the period T_0 as an example, as shown in Fig. 5(a), in the time domain, we have

$$S(t;T_0)=\begin{cases}1, & t\in\left[0,\frac{T_0}{2}\right)+kT_0\\ -1, & t\in\left[\frac{T_0}{2},T_0\right)+kT_0\end{cases};k\in\mathbb{Z}$$

and $S(t;T_0)=S(t+T_0;T_0)$. If T_0 is a period, kT_0 is also the period of $S(t;T_0)$.

If we apply DFT to a sequence sampled from $S(t;T_0)$, we can only obtain information at the frequencies $\omega\geq\omega_0=1/T_0$ in the frequency domain. For example, we can observe peaks in the DFT-spectrum at $k\omega_0$ and $k\in\mathbb{N}^+$, but we know nothing for frequencies with $\omega<\omega_0$. In contrast, when we apply TSA to a sequence sampled from $S(t;T_0)$, we can obtain the information in the period range of $T\geq T_0$ in the period domain. For example, we can observe peaks in the TSA-spectrum at kT_0 and $k\in\mathbb{N}^+$. Furthermore, an interesting finding is that for some specific cases, we can also obtain information for the period range $T<T_0$, or more exactly, for those T values satisfying $(T,T_0)=d>1$, which can be explained by Proposition 1 given in Section 2.2. In conclusion, DFT is more powerful in dealing with high-frequency (short-period) components but is helpless for low-frequency (long-period) components where TSA makes up for the weakness of DFT.

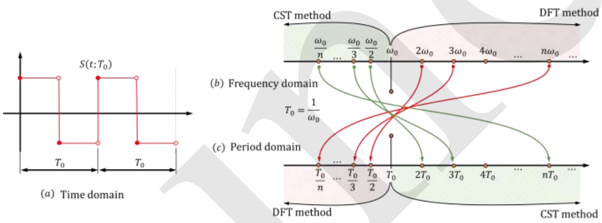


Fig. 1 Comparison between TSA and DFT in the frequency and period domains

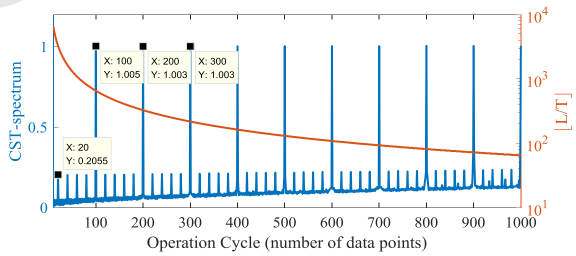
It should be noted that TSA can be shown to be powerful when comparing performances of TSA and DFT in two cases: (1) a rectangular signal with only one period, 100, denoted as $y_1(t)$, and (2) a signal combining two rectangular functions with periods 100 and 150, denoted as $y_2(t)$.

$$y_1(t)=S(t;100)+e(t)$$

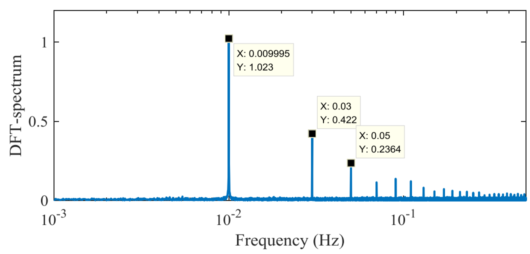
$$y_2(t)=0.5\cdot S(t;100)+0.5\cdot S(t;150)+e(t)$$

For both cases, a noise term $e(t)$ is added. It is white noise with a zero mean and a standard deviation of 1. For both cases, we sample $2^{16}(=65536)$ data points with the sampling frequency $F_s=1$. Then two discrete sequences, $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$ from $y_1(t)$ and $y_2(t)$ respectively, are obtained. Lastly, TSA and DFT are applied to both $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$. The results of the TSA-spectrum and the DFT-spectrum are presented in Fig. 2.

It can be seen in Fig. 2(a) that there are obvious peaks for operation cycles $T=100k$ and $k\in\mathbb{N}^+$. Furthermore, we can also observe small peaks at $T=20k$ and $k\in\mathbb{N}^+$. In contrast, in Fig. 2(b), we can observe peaks at $\omega=(2k-1)/100$ and $k\in\mathbb{N}^+$ in the frequency domain. However, the DFT-spectrum provides little information for $\omega<0.01$. In the result presented in Fig. 2(c), the major peaks of the TSA-spectrum are located at 100, 150, 200, 300, …, which shows the interaction between these two rectangular functions. It should be noted that there is a newly generated period of 300, which is the least common multiple of 100 and 150. Moreover, we can find small peaks for $T<100$ in the period domain. In contrast, we can observe peaks at $\omega=(2k-1)/100$ and $k\in\mathbb{N}^+$ and at $\omega=(2k-1)/150$ and $k\in\mathbb{N}^+$ in the frequency domain in Fig. 2(d), but no information is provided for $\omega<1/150$.



(a)



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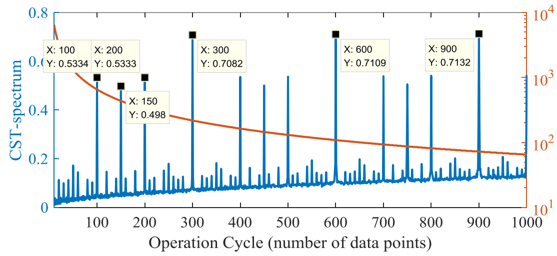
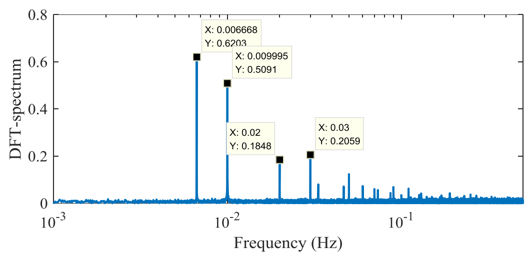


(c)



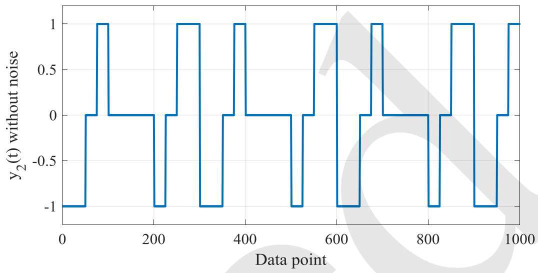
(d)

Fig. 2 Comparison between the TSA-spectrum and DFT-spectrum. (a) TSA-spectrum of Y_1; (b) DFT-spectrum of Y_1; (c) TSA-spectrum of Y_2; (d) DFT-spectrum of Y_2.

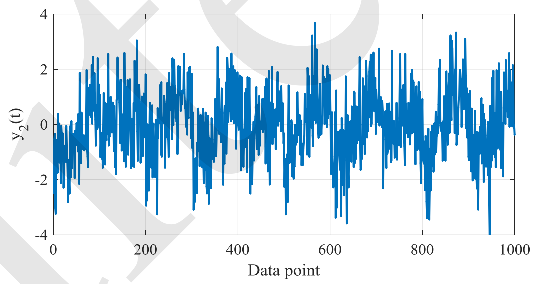
Fig. 3 provides a crucial demonstration of the practical advantages of TSA (Time-Synchronous Averaging) over DFT (Discrete Fourier Transform) in scenarios involving signals with low Signal-to-Noise Ratios (SNR). The physical interpretation of Fig. 3 is as follows:

In Fig. 3(a), we initially encounter a noise-free signal. Taking the sequence sampled from $y_2(t)$ as an example it can be found that the maximal magnitude of $\boldsymbol{Y}_2$ without noise is 1, which is equal to the standard deviation of the white noise. However, as noise is introduced in Fig. 3(b), the original waveform becomes nearly indistinguishable due to the low SNR.

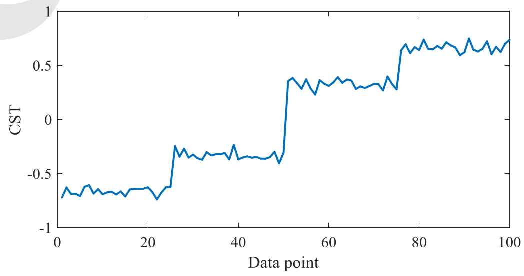
TSA comes to the forefront as a powerful solution to this challenge. Fig. 3(c) showcases the TSA vector when the operation cycle is set to 100. This specific choice effectively extracts the underlying periodic components, thus revealing the original signal, even in the presence of substantial noise. Fig. 3(d) unveils another facet of TSA. Here, the operation cycle is configured at 150, resulting in the extraction of a single period of $0.5 \cdot S(t; 150)$. Importantly, the result in Fig. 3(c) is not merely a singular period of $0.5 \cdot S(t; 100)$. Instead, it represents a blend of one period of and the TSA outcome of $0.5 \cdot S(t; 150)$, utilizing an operation cycle of 150. This phenomenon, known as the Period Aliasing Phenomenon (PAP), occurs when a signal encompasses two periodic components with periods T_1 and T_2 satisfying $(T_1, T_2) = d > 1$.



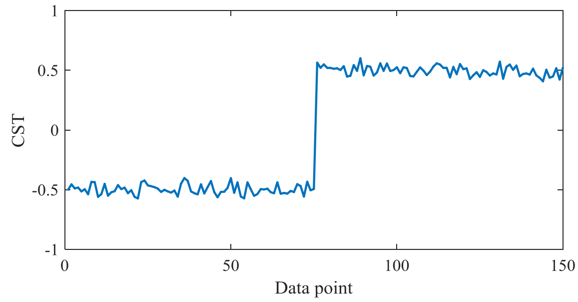
(a)



(b)



(c)



(d)

Fig. 3 The signal extraction performance of TSA. (a) Y_2 without noise; (b) Y_2 with noise; (c) the TSA vector with

the operation cycle set to 100; (d) the TSA vector with the operation cycle set to 150.

In summary, Fig. 3 effectively illustrates how TSA triumphs in extracting vital periodic components from noisy signals. This visual representation underscores TSA's prowess in addressing the challenges posed by low SNR scenarios. Furthermore, it highlights the occurrence of the Period Aliasing Phenomenon (PAP) as a critical aspect of TSA's signal processing capability, ultimately enhancing signal extraction, even when confronted with challenging noise levels.

3 Application to the bearing test dataset

To illustrate the performance of TSA, this section presents an application of TSA for bearing-fault diagnosis. The vibration of a rotating bearing provides a stable quasiperiodic signal which can be a good example of the advantages of TSA. The dataset used in this section is the widely used Bearing Test Dataset published by Case Western Reserve University (CWRU) (Bearing data center, September, 2019).

3.1 Problem description

Bearings are one of the greatest inventions of mankind. As a type of typical rotating machinery, bearings are widely used in all kinds of vehicles, trains, wind turbines, engines, etc (Li et al., 2018; Teng et al., 2017; Lin et al., 2018). Therefore, it is of great importance to develop accurate and robust bearing-fault diagnosis techniques to keep machines safe and reliable. In this section, we take a typical rolling bearing 6205-2RS JEM SKF as an example. The structure of Bearing 6205 consists of an inner raceway, an outer raceway, and a cage train with multiple rolling elements, as illustrated in Fig. 4.

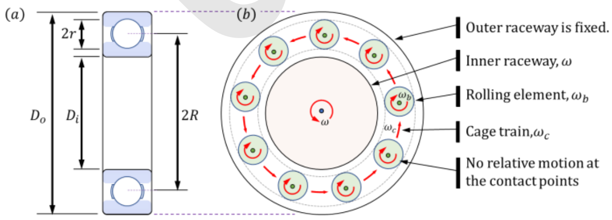


Fig. 4 Illustration of the structure of Bearing 6205-2RS JEM SKF. 2R=39.03 mm, 2r= 7.94 mm, D_o=52 mm, D_i=25 mm. There are 9 rolling elements.

In practice, the vibration of a rotating bearing is a typical quasiperiodic signal. A perfect bearing helps the inner shaft rotate freely and smoothly. Unfortunately, it can fail due to mechanical damage, crack, wear damage, etc. When a failure occurs, a series of high-level short pulses in acceleration can be observed, owing to the ball passing over the defect and causing the bearing to vibrate abnormally. For example, as illustrated in Fig. 5, there is a defect on the inner raceway. The vibration of the bearing will contain a short-period signal caused by the ball hitting the defect between points B and A. The period of the fault signal is related to the diameters 2R and 2r and to the angular speed of the shaft.

To address the performance of TSA, we transfer data from the CWRU dataset (Bearing data center, September, 2019) to the TSA-spectrum through a multi-step process. Information on these four data samples is presented in Table 1. Each data sample is collected in an independent test process. Initially, we collect vibration data from the CWRU dataset, encompassing signals from various bearing conditions and machinery types. Subsequently, we apply data preprocessing techniques such as cleaning, noise reduction, and resampling to ensure data uniformity. The pivotal step involves employing Time Synchronous Averaging (TSA) to align periodic components within the data, followed by the calculation of the TSA-spectrum. From the TSA-spectrum, we extract pertinent features that capture the distinctive characteristics of periodic components. These extracted features serve as valuable inputs for machine learning models employed in bearing condition classification and fault detection. The computational overhead of this data transfer process varies depending on the dataset size and the available computational resources. To enhance efficiency, we explore optimization strategies, including parallel computing and judicious feature selection. This comprehensive data transformation process is an integral part of our research, facilitating the accurate and effective identification of bearing-faults for predictive maintenance and machinery reliability improvement.

Additionally, since there is no relative motion at the contact points between the rolling elements and the inner or outer raceway, the relationship between the angular speeds ω and ω_c is given as $\omega_c/\omega = (R-r)/R/2$. For Bearing 6205, we have

$\omega_c/\omega \approx 0.4$, indicating that for every 2.5 cycles of the inner race, the cage train moves exactly 1 cycle.

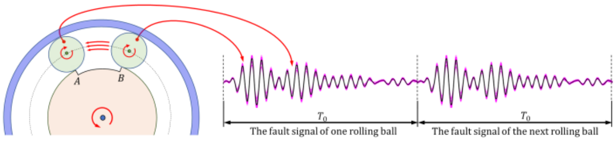


Fig. 5 The hidden periods in the vibration signal of a rotating bearing with defects.

Table 1 Four data samples from the Seeded Fault Test Data published by CWRU.

File ID	Sample number	Fault type	Operating condition	Fault size
1	100	Normal	1725 r/min, 0 HP	
2	122	Ball failure	1796 r/min, 0 HP	
3	109	Inner raceway failure	1796 r/min, 0 HP	0.007 inches
4	135	Outer raceway failure centered to load	1797 r/min, 0 HP	

3.2 Results of TSA

A. Major cycle analysis using TSA-spectrum

First, we apply TSA to four different types of bearing signals and calculate their TSA-spectra, as presented in Fig. 6. It should be noted that the File IDs in Table 1 correspond to the sub-figures in Fig. 6. The TSA-spectrum can reveal the cycles hidden inside a measured sequence. Note that since the length of the original data piece and the rotation velocity of the bearing are different from each other, the maximal value of the sample number, namely, the maximum of $\lfloor L/T \rfloor$ in the right axis of each subfigure, varies for different measured data. By comparing the four TSA-spectra in Fig. 6, the following conclusions can be drawn:

(1) In Fig. 6(a), all peaks show similar values, and the values of the TSA-spectrum are smaller than those in Fig. (b)-(d). This suggests that the bearing condition represented in Fig. 6(a) is relatively more stable, with less variance in its signal.

(2) Except Fig. 6(a), all other TSA-spectra show obvious peaks approximately 2.5 MC and 5.0 MC. These peaks indicate prominent cycle repetitions at these intervals, hinting at specific bearing conditions that recur with each rotation.

(3) In Fig. 6(b), associated with a bearing experiencing ball failure, a pronounced peak is evident at 1 MC. This peak stands as a distinctive signature of ball failure, denoting the defect's interaction with other bearing components during each rotation. Moreover, Fig. 6(c) manifests a notable peak at the operation cycle of 5, which is indicative of "Inner raceway failure." An inner raceway defect leads to a high-frequency impact each time the ball traverses the fault in its revolution, which is prominently represented as a distinct peak in the TSA-spectrum.

Combining the parameters of Bearing 6205, for each turn of the rotating shaft, the cage train (and all rolling elements) moves 0.4 turns. This indicates that with every 2.5 turns of the rotating shaft, all rolling elements complete one full rotation and, with every 5 turns, the rolling elements make two full rotations. As a result, every 5 turns will reset all conditions of both the rotating shaft and rolling elements (and cage train), signifying that the basic cycle is 5 MC. Particularly, we focus on the TSA-spectrum with operation cycles close to 5 MC, as it represents the primary resetting point for bearing elements, making it essential for understanding bearing health.

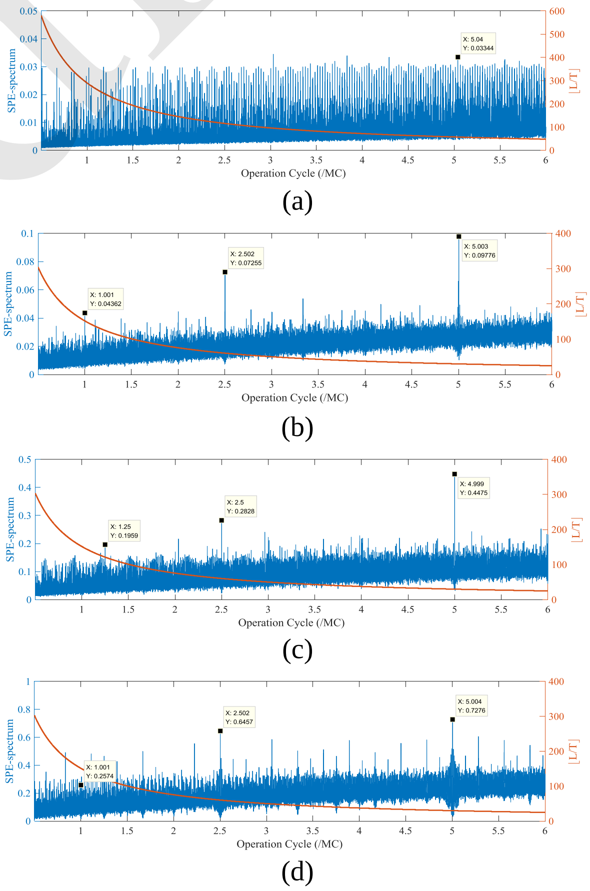


Fig. 6 Comparison between the TSA-spectra of the four

different signal types when a 48k*3 sampling frequency is applied. The unit MC means the number of data points for one cycle of the rotation shaft.

B. Fault signal extraction

By exploring the TSA-spectra, we can obtain the precise basic cycle of a given signal. Then, we can directly apply the TSA to the signal with the precise operation cycle and obtain the denoised signal of our interest.

Taking the fourth data sample in Table 1 as an example, the result of TSA is illustrated in Fig. 7(a). The shifted raw data is presented in cyan, while the black curve is the TSA vector, namely, the averaged result of the shifted raw data. The operation cycle is 167.171 (ms), which is the time required for 5 turns of the rotating shaft. It can be observed that there are 18 obvious pulse-like signals. Considering that there are 9 rolling elements of Bearing 6205, the 18 pulse-like signals can be divided into two groups. The first 9 pulses, namely, (1)~(9) in Fig. 7(a), are the first time the 9 rolling elements run across the defect area on the outer race, while the other 9 pulses, (1')~(9'), relate to the second time the rolling elements run across the defect area. Note that the pulses (i) and (i') are actually the signals of the same rolling ball running across the defect area. However, there is a 180° phase difference on the inner race between pulse (i) and (i'), since every 2.5 turns of the rotating shaft lead to a 180° phase change on the inner race.

Let us focus on the waveform of pulse (3). We zoom in on the x-axis as presented in Fig. 7(b). It can be observed that the shifted raw data consists of a series of similar waveforms with small phase differences. Particularly, we visualize the shifted raw data in an image style, as shown in Fig. 7(c), where the x-, y- and z-axes are time, the sample number and the magnitude of acceleration, respectively. Fig. 7(c) shows a periodic change in phase space, at approximately 0.12 (ms) for every 5 turns of the rotating shaft. The phase changing phenomenon is caused by the unstable rotating speed of the driving motor, which will lead to an additional smoothing effect on the TSA vector.

Additionally, we can estimate the phase differences between every two sampling cycles of the shifted raw data by using a correlation function. Following a phase shift operation, we obtain the result of aligned raw data and a corrected TSA vector, as illustrated in Fig. 7(d), where the lines in magenta are the aligned raw data within one major cycle. Similarly, as shown in Fig. 7(c), we also visualize the aligned raw data in an image style as displayed in Fig. 7(e), where the parallel stripes indicate that the variance of rotation speed is reduced. To address the performance of the alignment process, we calculate the standard deviation of the shifted raw data and aligned raw data with respect to different sample numbers, namely, along the y-axis of Fig. 7(c) and (d). The standard deviation of the aligned raw data is reduced to 0.16 from that of the shifted raw data, which is 0.72. The result in Fig. 7(d) shows a good repeatability of the acceleration waveform of a rotating bearing.

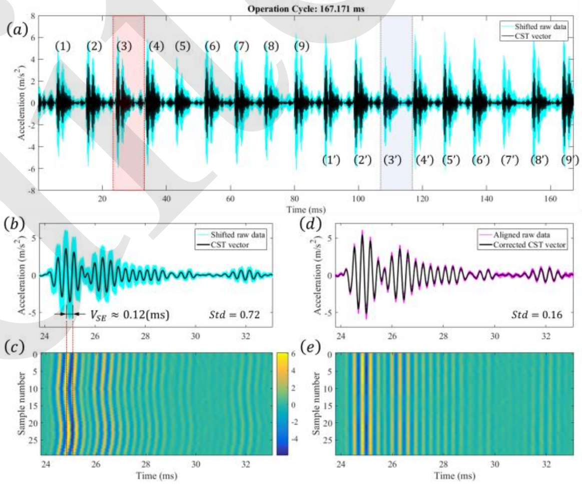


Fig. 7 The TSA of ORF_CE sample '135_1796.csv''. The result is given at an operation cycle of 167.171 ms. The smoothing effect $V_{SE} \approx 0.12$ ms.

C. Extracted fault signal comparison

By applying the TSA method to the four data samples presented in Table 1, we can extract the featured waveform within one entire cycle of each case. The results are presented in Fig. 8. The following points can be drawn from the results in Fig. 8:

(1) For a normal bearing without defects, as presented in Fig. 8(a) and (b), the TSA vector over the operation cycle of 5 MC is a stationary waveform consisting of two sinusoidal signals with cycles of approximately 2.78 (ms) and 0.12 (ms), respectively. These two sinusoidal signals relate to the natural vibration period of the test apparatus.

(2) In Fig. 8(c) and (d), the BF case, the TSA

vector contains information on both the rotation of the cage train and the self-rotation of the rolling elements. Each hit between the defect area on the rolling elements and the contact point on the inner race or outer race will cause a hitting signal. Since the hitting frequency is higher than the vibration damping speed, different hitting signals overlap together, and it is difficult to find the pattern of a single hit.

(3) In Fig. 8(e), the IRF case, we can observe approximately 27 pulse-like patterns within the entire cycle. Similarly, in Fig. 8(g), the ORF_OE case, there are 18 significant pulse-like patterns. Both fault types are related to the number of rolling elements in the cage train, which is 9 for Bearing 6205. Consequently, we know that within a basic cycle of 5 MC, each ball hits the defect area 3 times for the inner race and 2 times for the outer race.

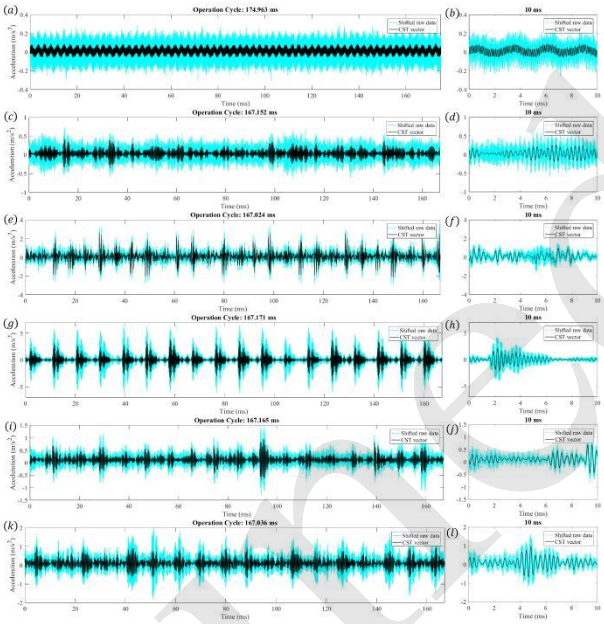


Fig. 8 Extracted waveforms of different fault types. (a), (c), (e), and (g) relate to file ID 1 to 4 in Table 1. (b), (d), (f), and (h) are enlarged views of the first 10 ms of the waveform.

4 Conclusions

In this paper, a TSA-spectrum based on super-resolution analysis is proposed for visualizing possible periodic components of a signal in a period domain. To reveal the mathematical properties regarding its performance on a quasiperiodic signal, we proposed and proved three propositions. In a case study, TSA has shown a great advantage in processing the vibration signal of a bearing with a defect. From the application level, further effort can be made toward applying TSA in other fields with problems of QSP. At the theoretical level, we can broaden our studies to reveal more about the period aliasing phenomenon (PAP). There is great potential in taking advantage of the PAP to reconstruct hidden periodic components, even if the periods share common dividers.

The main findings are summarized as follows:

(1) The theoretical framework of TSA with essential definitions and propositions is proposed to reveal the mathematical properties regarding its performance in analyzing the periodic components hidden in a quasiperiodic signal. Super-resolution analysis is also introduced to enhance the performance of the TSA-spectrum.

(2) The relationship and difference between the DFT and TSA are discussed in detail. It is demonstrated that TSA-spectrum can present more details of the cross effects between different periodic components of a quasiperiodic signal. TSA can be an important supplement to DFT in the long-period (low-frequency) range.

(3) A case study is performed to illustrate the good performance of TSA for bearing-fault diagnosis. The major cycle analysis for four different types of bearing signals using the TSA-spectrum is presented. Then the TSA is applied to the signal with the precise operation cycle and the denoised signal of the relevant interested zone is obtained. The extracted fault signals for four data samples are presented. The results show that TSA has a great advantage in processing the vibration signal of a bearing with a defect.

(4) TSA could be extended to reveal information about the PAP, especially for applications involving quasiperiodic signal processing.

In summary, our study introduces and demonstrates the effectiveness of the TSA-spectrum and super-resolution analysis in the context of Bearing-fault Signal Identification. These techniques enable the precise extraction and analysis of periodic components within bearing signals, even in challenging scenarios involving noise and changing periodicities. This enhancement significantly contributes to the reliability and accuracy of bearing-fault identification, a crucial aspect of predictive maintenance, and to the overall improvement of machinery and industrial system reliability. By incorporating these advances, we pave the way for more efficient and effective fault diagnosis in the field of bearing health monitoring.

Acknowledgments

The present work has been supported by the National Natural Science Foundation of China (52008198, 51425804), the National Natural Science Foundation of China (U20A20283), the National Natural Science Foundation of China (U1813222), the Shenzhen International Cooperation Research Program (GJHZ20200731095009029), the Department of Science and Technology of Guangdong Province (No. 2019TQ05Z654).

Author contributions

Wei Feng designed the research. Yuan Wang and Huiyue Tang processed the corresponding data. Zengle Ren wrote the first draft of the manuscript. Xinan Chen helped to organize the manuscript. Zengle Ren revised and edited the final version.

Conflict of interest

Zengle Ren, Yuan Wang, Huiyue Tang, Xinan Chen and Wei Feng declare that they have no conflict of interest.

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中文概要

题　　目：基于超分辨率分析的同步平均频谱及其在轴承故障信号识别中的应用

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目　　的：精密冲压工艺过程中环境变量的波动导致工件出现破裂和皱褶等缺陷。本文旨在探讨精密冲压工艺过程中环境变量（工件材质、冲压速度、压力和温度变化等）对冲压质量的影响，研究适应性工艺设计方法，以提高精密冲压工件的质量。

创新点：1. 通过马尔科夫模型方程，推导出环境变量与精密加工波动公差之间的关系；2. 建立试验模型，成功模拟适应性冲压工艺过程。

方　　法：1. 通过实验分析，推导出冲压过程中的晶粒流动和强度变化对成型 零件的尺寸公差波动产生较大的影响（图 2 和 3）；2. 通过理论推导，构建环境变量与加工波动公差之间的关系，得到适应性的工艺参数调节方案（公式(6)）；3. 通过仿真模拟，运用适应性设计方法在精密冲压过程中对工艺参数进行适应性调节，验证所提方法的可行性和有效性（图 5）。

结　　论：1. 精密冲压过程中工艺参数需要根据不同的环境变量进行调节；2. 环境变量与加工波动公差之间存在映射关系，运用隐马尔科夫模型实现关联表征；3. 运用适应性设计方法对精密冲压工艺参数进行调节，加工波动公差明显减小，工件质量得到提高。

关键词：时间同步平均；频谱；准周期信号处理；超分辨分析；轴承故障检测