Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering) in press
www.jzus.zju.edu.cn; www.springer.com/journal/11582
E-mail: jzus_a@zju.edu.cn

Correspondence

https://doi.org/10.1631/jzus.A2200525

Two-dimensional plane-strain consolidation for unsaturated soils under non-uniform trapezoidal loads

Chengjia TANG[1], Lei WANG[1✉], Sidong SHEN[1], Minjie WEN[2], Annan ZHOU[3]

[1]School of Urban Railway Transportation, Shanghai University of Engineering Science, Shanghai 201620, China
[2]MOE Key Laboratory of Soft Soils and Geoenvironmental Engineering, Zhejiang University, Hangzhou 310058, China
[3]School of Civil, Environmental and Chemical Engineering, Royal Melbourne Institute of Technology (RMIT), Melbourne VIC 3001, Australia

1 Introduction

In highway construction, filled embankments are trapezoidal, and the ground is always improved by sand wells or columns. During embankment construction, because the width and height of the embankment are changing, a non-uniform load that varies with time and lateral location is applied to the underlying ground. The consolidation phenomenon under two-dimensional (2D) conditions will keep pace with the construction of the embankment. In addition, because of evaporation and rainfall, the soils are mostly unsaturated. Therefore, it is meaningful to research the consolidation properties of unsaturated ground under non-uniform loading.

Terzaghi's (1943) consolidation theory has been remarkably successful in predicting many types of soil settlement and has been the strongest driving force in establishing the field of geotechnics. From a mathematical-physical point of view, Biot (1941) proposed two possible extensions of Terzaghi's theory: extending it to the three-dimensional case, and establishing equations valid for any arbitrary load variable that varies with time. The physical properties of unsaturated soil, a material with three phases, are extremely complex. Scott (1963) derived the consolidation equations for unsaturated soils containing air bubbles, which were based on variation in saturation and pore ratio. Barden (1965) first proposed the classification of unsaturated soils into three categories based on the degree of soil saturation. Fredlund and Morgenstern (1977) proposed a stress-state variable suitable for unsaturated soils. Fredlund and Hasan (1979) proposed the one-dimensional consolidation theory for unsaturated soils based on the assumption that the air phase was continuous. The one-dimensional consolidation theory was soon extended to the two-dimensional case (Dakshanamurthy and Fredlund, 1980). Almost at the same time, a generalized mathematical model of unsaturated soil consolidation was established by Lloret and Alons (1981). Chang and Duncan (1983) proposed an empirical formula for the permeability of pore fluids. Fredlund et al. (1998) ignored the meteorological continuum in their numerical study of consolidation and instead used the atmospheric pressure as the pore air pressure. Ausilio and Conte (1999) defined the average consolidation degree and combined it with the consolidation rate. Conte (2004) transformed the multi-dimensional consolidation problem into a one-dimensional (1D) consolidation problem based on the results of Fredlund and his coworkers (1979; 1980).

In recent years, boundary conditions and external loads have been the focus of research. Qin and Sun (2008; 2010a; 2010b; 2014) gave the analytical and semi-analytical solutions for one-dimensional consolidation of unsaturated soil at finite thickness, and investigated the effects of exponential loading and free drainage wells on one-dimensional consolidation. Shan (2013) studied the effect of mixed

✉ Lei WANG, wangleiwangjiang@163.com
Lei WANG, https://orcid.org/0000-0001-9423-7866

Received Nov. 4, 2022; Revision accepted May 11, 2023; Crosschecked

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boundaries on 2D plane-strain consolidation. Ho and Fatahi (2014; 2015,2016; 2020) studied the effect of several time-dependent external loads on the consolidation of unsaturated soils. Wang et al. obtained a more general semi-analytic solution for two-dimensional consolidation of unsaturated soil by Laplace transform. In addition, they obtained the semi-analytic solutions for different drainage boundaries. The consolidation behavior of three time-dependent loads was also investigated (2017a; Wang et al., 2017b; 2018; 2019). In the parametric study of 1D consolidation or 2D consolidation of unsaturated soils, the permeability coefficient and load function are often among the constituent parameters studied.

In general, the studies carried out so far based on Fredlund's (1979) consolidation theory have dealt with uniform external loads, and the focus has generally been on load variation with time. However, as explained above, the load was homogeneous and time-dependent; few studies have considered non-uniform loads, such as the trapezoidal load that is induced by construction of an embankment. There is a need for engineers and researchers to fill this gap.

In this study, we chose a trapezoidal load as a special case of non-uniform load in order to make calculation more convenient and also for its similarity to the actual roadbed load. Based on the original reference model of 2D plane consolidation under uniform loading, we established a specific model under non-uniform loading conditions. Then, we incorporated the non-uniform trapezoidal loads into the mathematical model. Based on the semi-analytic solution of 2D consolidation, we derived the semi-analytic solution considering external loading with width and time, using Laplace transform and Fourier sine series expansion. During the construction of a trapezoidal embankment, the height and slope vary under different construction conditions and non-uniform loads may lead to uneven deformation of the underlying unsaturated soil. Therefore, we chose the height and slope of the trapezoidal load as the main parameters for analysis to study their effects on pore pressure and settlement. The hope was that this would not only further improve the consolidation theory of unsaturated soils, but also provide some theoretical basis for dealing with unsaturated soils in highway engineering.

2 Mathematical model

2.1 Mathematical model and external load

Based on the 2D plane-strain consolidation theory (Dakshanamurthy and Fredlund, 1980), we designed a reference model for 2D plane-strain consolidation of unsaturated soils with a non-uniform load, as shown in Fig. 1. The non-uniform load $Q(x,t)$ considered here is simplified as a trapezoidal distribution in the x-direction, which also changes with time. The soil thickness is h, and the distance of the vertical drains is l.

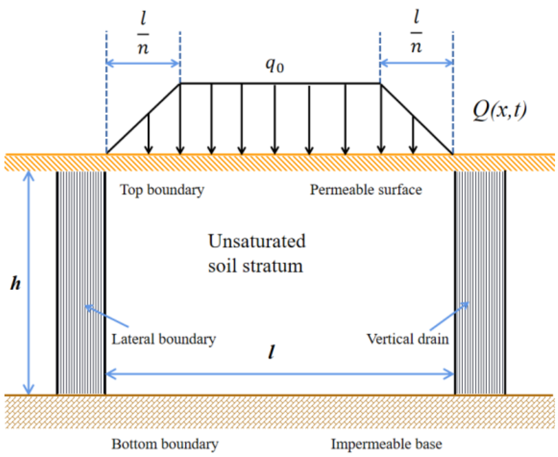

Fig. 1. Mathematical model of 2D plane-strain consolidation of unsaturated soil under a non-uniform load.

The function of the external load $q(x,t)$ is related to both time t and distance x:

$$q(x,t) = \mathrm{f}(x)\mathrm{g}(t) \tag{1}$$

where

$$\mathrm{g}(t)=\begin{cases} at, t < t_0 \\ 1, t \geq t_0 \end{cases} \tag{2}$$

$$\mathrm{f}(x) = \begin{cases} x\,\frac{nq_0}{l},\ 0 < x < \frac{l}{n} \\ q_0,\ \frac{l}{n} < x < \frac{(n-1)l}{n} \\ nq_0 - x\,\frac{nq_0}{l},\ \frac{(n-1)l}{n} < x < l \end{cases} \tag{3}$$

n is a positional parameter of the load. l/n is the width of the road shoulder. q_0 is the load magnitude, which represents the embankment height.

The schematic diagram of the external load

$q(x,t)$ is included in Section S1 of the electronic supplementary materials (ESM).

2.2 Basic assumptions

To avoid the influence of some secondary factors on analysis of the main factors, the reference model was established with the following basic assumptions: (1) The soil is homogeneous; all parts of the soil have the same properties; (2) During consolidation, the water and air phases are independent and continuous; (3) The soil grain and pore water cannot be compressed; (4) Movement of water vapor due to temperature variation and diffusion of air in water are not considered; (5) The coefficients of consolidation and permeability are constant values; (6) Deformation of unsaturated soil happens along the lateral direction (*x*-direction).

We fully allow that the permeability coefficient of the water phase will change during the consolidation process in unsaturated soils, i.e., the coefficient of permeability is a function of any two of the three parameters: *e*, *S*, *w*. That is, $k_w=f_1(e,w)$; or $k_w=f_2(s,e)$; or $k_w=f_3(s,w)$. However, because of the air phase, deriving solutions of excess pore-air and pore-water pressures in the consolidation process in unsaturated soils is a great challenge, considering the variations in the permeability coefficient. It might be acceptable to assume that the permeability coefficient of water is constant during the transient process for a particular stress increment.

2.3 Governing equations

After considering load action, the governing equations are as follows (Dakshanamurthy and Fredlund, 1980):

$$\frac{\partial u_{\mathrm{a}}}{\partial \mathrm{t}}=-C_{\mathrm{a}}\frac{\partial u_{\mathrm{w}}}{\partial \mathrm{t}}-C_{\mathrm{v_x}}^{\mathrm{a}}\frac{\partial^2 u_{\mathrm{a}}}{\partial \mathrm{x}^2}-C_{\mathrm{v_z}}^{\mathrm{a}}\frac{\partial^2 u_{\mathrm{a}}}{\partial \mathrm{z}^2}+C_{\sigma}^{\mathrm{a}}\frac{\partial \sigma_{\mathrm{z}}}{\partial \mathrm{t}} \quad (4)$$

$$\frac{\partial u_{\mathrm{w}}}{\partial \mathrm{t}}=-C_{\mathrm{w}}\frac{\partial u_{\mathrm{a}}}{\partial \mathrm{t}}-C_{\mathrm{v_x}}^{\mathrm{w}}\frac{\partial^2 u_{\mathrm{w}}}{\partial \mathrm{x}^2}-C_{\mathrm{v_z}}^{\mathrm{w}}\frac{\partial^2 u_{\mathrm{w}}}{\partial \mathrm{z}^2}+C_{\sigma}^{w}\frac{\partial \sigma_{\mathrm{z}}}{\partial \mathrm{t}} \quad (5)$$

where u_{a} and u_{w} are excess pore-air and pore-water pressures; C_{a} and C_{w} are interactive constants with regard to the water and air phases; C_{σ}^{a}, $C_{\mathrm{v_x}}^{\mathrm{a}}$, and $C_{\mathrm{v_z}}^{\mathrm{a}}$ are the solidification coefficients related to the air phase; C_{σ}^{w}, $C_{\mathrm{v_x}}^{\mathrm{w}}$, and $C_{\mathrm{v_z}}^{\mathrm{w}}$ are the solidification coefficients related to the water phase; and σ_z are the total stresses in the vertical direction. The consolidation parameters are given in Section S2 of the ESM.

The coefficients of volume change and the permeability coefficients for the air and water phases can be drawn from the 2D-plane consolidation theory for unsaturated soils proposed by Dakshanamurthy and Fredlund (1980). Ng et al. (2002) proposed a new, simple system for accurately measuring overall total volume changes in unsaturated soil specimens with a triaxial apparatus. The measuring system was reasonably linear, reversible, and repeatable, making it a powerful tool for obtaining the coefficients of volume change. Šimůnek et al (1998) discussed three field methods for estimating soil hydraulic properties by numerical inversion of Richards' equation, which help in obtaining the permeability coefficients.

2.4 Boundary conditions

In the mathematical model, it is clear that the top boundary is in direct contact with the air and is therefore fully permeable; in contrast, the bottom boundary is impermeable. The lateral sand wells are also fully permeable. The expressions of the boundary conditions are in Section S3 of the ESM.

3 Semi-analytical solutions

According to the lateral boundary, the variables Z in the governing equations are separated out by the Fourier series. The semi-analytic solution of the governing equation can be obtained as Eqs. (6) and (7) by Laplace transform. The specific solution procedure is given in Section S4 of the ESM.

$$u_{\mathrm{a}}(x,z,t)=L^{-1}[\textstyle\sum_{k=1}^{\infty}\widetilde{U}_{\mathrm{a}}(x,s)\sin(Kz)] \quad (6)$$

$$u_{\mathrm{w}}(x,z,t)=L^{-1}[\textstyle\sum_{k=1}^{\infty}\widetilde{U}_{\mathrm{w}}(x,s)\sin(Kz)] \quad (7)$$

where $U_{\mathrm{a}}(x,t)$ and $U_{\mathrm{w}}(x,t)$ are the generalized Fourier coefficients of the variation for the air and water phases with time *t*, respectively, and $K=\frac{(2k+1)\pi}{2h}$, k=1, 2, ….

The basic equation for the 2D plane strain is

$$\tilde{\varepsilon}_{\mathrm{v}} = (m_2^{\mathrm{s}}-2m_1^{\mathrm{s}})\tilde{u}_{\mathrm{a}}-m_2^{\mathrm{s}}\tilde{u}_{\mathrm{w}} + \frac{m_2^{\mathrm{s}}u_{\mathrm{w}}^0-(m_2^{\mathrm{s}}-2m_1^{\mathrm{s}})u_{\mathrm{a}}^0+m_1^{\mathrm{s}}Q(x,s)}{s} \quad (8)$$

where ε_{v} is the volume strain, $m_1^s=m_1^{\mathrm{a}}+m_1^{\mathrm{w}}$,

$m_2^{\mathrm{s}} = m_2^{\mathrm{a}} + m_2^{\mathrm{w}}$.

Eq. (10) is integrated in the z-direction to obtain the solution for unsaturated soil sedimentation in the Laplace domain, as follows:

$$\tilde{w}(s) = \int_0^h \tilde{\varepsilon}_{\mathrm{v}} \mathrm{d}z \tag{9}$$

Semi-analytical solutions of the 2D plane-strain consolidation problem for unsaturated soils under non-uniform loads are Eqs. (6), (7), and (9). Then one can apply Crump's method to complete the Laplace inversion and obtain the corresponding solution in the time domain.

4 Validation

During validation, the solution under non-uniform loading conditions is degraded to the solution under exponential loading varying with time. The development curves of the relative settlement (w*) and the expressions of the exponential loads are presented in Fig. 2. The values of the main physical parameters are as follows: H=5m, L=2m, $n_0 = 50\%$, $S_0 = 80\%$, $m_1^{\mathrm{s}} = -2.5\times10^{-4}\ \mathrm{kPa}^{-1}$, $m_2^{\mathrm{s}}/m_1^{\mathrm{s}} = 0.4$, $m_1^{\mathrm{w}}/m_1^{\mathrm{s}} = 0.2$, $m_2^{\mathrm{w}}/m_1^{\mathrm{s}} = 4$, $u_{\mathrm{a}}^0 = 20$ kPa, u_{w}^0 =40 kPa, $k_{\mathrm{w}} = 10^{-10}$ m/s, $R = 8.314\mathrm{J\ mol^{-1}K^{-1}}$; $M = 0.029\mathrm{kg\ mol^{-1}}$; $\Theta = (\theta^0 + 273.16)$K; and $\theta^0 = 20°$C. In this comparison, $\frac{k_{\mathrm{a}}}{k_{\mathrm{w}}} = 10^2, 10, and\ 1$ are adopted. The solid and dashed lines reflect the calculated results of the degraded solution and the existing one (Ho et al., 2015), respectively. Note that the solid and dashed lines are aligned. Therefore, the solution after degradation is consistent with the solution in the literature. The correctness and validity of the semi-analytic solutions obtained are also proved.

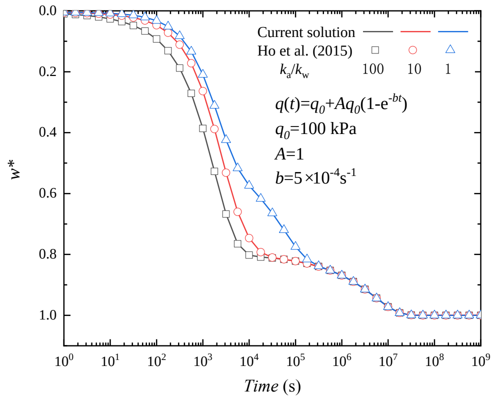

Fig. 2. Curves of relative settlement with different ratios of $k_{\mathrm{a}}/k_{\mathrm{w}}$ under exponential loading

5 Example analysis

The physical parameters needed for parametric analysis are listed in the previous section. Since the initial value of the external load is zero, $u_{\mathrm{a}}^0 = 0\ kPa$ and $u_{\mathrm{w}}^0 = 0\ kPa$. In the following work, the influence of calculation-point location, shoulder width, roadbed height, and roadbed shape on consolidation characteristics will be the focus.

In order to enrich the analysis, the reference models and some analysis for each section have been included in Section S5 of the ESM.

5.1 Consolidation at different calculated points

Fig. 3 demonstrates the variations in excess pore pressure considering different calculated points. Herein, $n = 4$, $q_0 = 100\ kPa$. Eqs. (1), (2), and (3) show that the trapezoidal load is increased linearly with time to the maximum value and then kept constant. The load reaches its maximum value for $t = 2\times10^3 s$. The excess pore pressure under the road shoulder is significantly less than that under the pavement; the closer the calculated point is to the pavement, the higher the extreme value of pore pressure. The maximum value of the pore pressure under the pavement is almost the same, as shown in Fig. 3. However, since the distance of the calculation point is different from the boundary, the rate of excess pore-pressure dissipation is different. The curves of pore-water pressure exhibit almost no plateau period under the road shoulder, but there is a significant plateau period under the pavement. The air pressure under the shoulder is relatively small, which also makes the plateau period of the curve under the shoulder relatively insignificant. The excess pore-air pressure under the pavement reaches an extreme value under load at $t = 10^4 s$, which also leads to the plateau period of curves under the pavement at the same time. After $t = 10^4 s$, the rate of excess pore-air-pressure dissipation is different because of different permeability conditions caused by different distances between the calculation point and the boundary, which also leads to different excess pore-water-pressure-profiles after the plateau period.

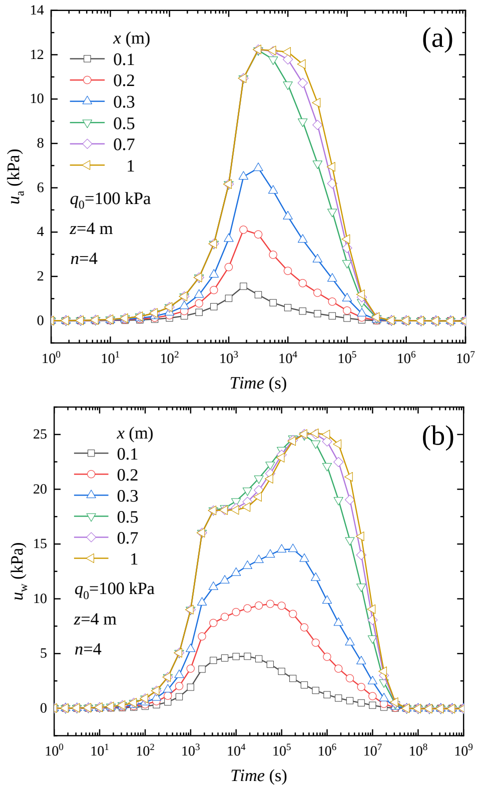

Fig. 3. Effect of different investigated widths x on the (a) excess pore-air pressure; and (b) excess pore-water pressure under non-uniform loading

5.2 Consolidation with different road-shoulder widths

The parameter n also implies different road-shoulder widths. The horizontal variations in excess pore pressure with different road-shoulder widths are depicted in Fig. 4. To make the variation in pore pressure due to different loads more obvious, we chose different moments when the excess pore pressure approaches its maximums for the water and air phases. The moment of pore-pressure maximum can be seen in Fig. 3. Since the pore pressure is caused by external load, the distribution of excess pore pressure in the horizontal direction is similar to that of external load in the horizontal direction. The distributions of excess pore-water and pore-air pressure are also quite similar. The greater the shoulder width, the smaller the pavement width, as the distance between the drainage wells is constant. It is easy to see from Fig. 4 that the distribution of excess pore pressure varies with the width of the road shoulder. Most of the pore pressure is concentrated at the pavement. As shown in Fig. 4, when narrow shoulders are adopted, the excess pore-pressure variation under non-uniform loading is closer to that seen under uniform loading. Since the values of q_0 are equal, the pore pressures under the pavement are almost the same. The variation in shoulder width only affects the pore pressure of the shoulder.

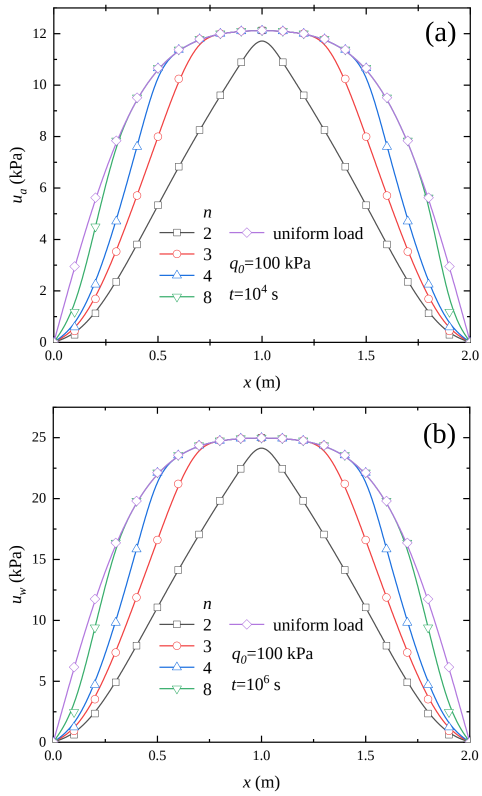

Fig. 4. Effect of different values of *n* on (a) excess pore-air pressure; and (b) excess pore-water pressure under non-uniform loads

5.3 Consolidation with different roadbed heights

With the extension of highways to mountainous areas, high-fill roadbeds have become the most common roadbed type in mountainous highway construction. Therefore, we investigated the influence of fill height on 2D-plane consolidation of unsaturated soils. The parameter q_0 represents roadbed height. Fig. 5 plots the variation in excess pore pressure under loads with different roadbed heights. The maximum pore-pressure value is marked in Fig. 5. The maximum excess pore-pressure value at $q_0 = 120kPa$ is 120% of that at $q_0 = 100kPa$. The same is true for $q_0 = 60kPa$ and $q_0 = 80kPa$.

Although the pore pressure under the shoulder increases proportionally with the increase in roadbed height, the pore pressure under the shoulder is originally much smaller than that under the road surface. Therefore, an increase in the height of the roadbed will lead to an increase in the difference between the pore pressure under the shoulder and that under the road surface.

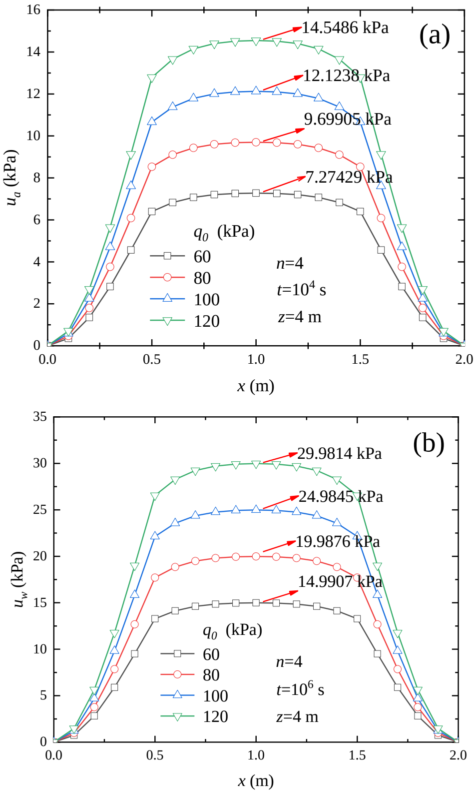

Fig. 5. Effect of different values of q_0 on (a) excess pore-air pressure; and (b) excess pore-water pressure under non-uniform loads

6 Conclusions

In this study, we chose a trapezoidal load as a special case of non-uniform load to study the effect of non-uniform load on consolidation behavior. Based on the 2D plane-strain consolidation theory for unsaturated soils proposed by Dakshanamurthy and Fredlund (1980), the semi-analytical solution of the 2D plane-strain consolidation problem for unsaturated soils considering loads varying along the horizontal direction was derived by Laplace transform and Fourier sine expansion. The solution under non-uniform loading conditions was degraded to the solution under exponential loading varying with time. The correctness and validity of the obtained solutions are demonstrated by verification.

The following conclusions can be obtained by analyzing the curves and 2D plots.

(1) The excess pore pressure under the road surface, especially on the embankment centerline, starts to dissipate late, but dissipates quickly. The excess pore air and water pressure dissipates from the top.

(2) Compared with the excess pore pressure under the road shoulder, that under the road surface is not only higher, but takes longer to completely dissipate. As a result, the settlement difference between the pavement and the shoulder will become larger and larger over time.

(3) An increase in fill height increases the final settlement at the pavement but has less impact on the road shoulder. Therefore, the difference in the settlement between pavement and road shoulder is larger in high-fill roadbeds. During the construction of the roadbed, the height of the roadbed should be controlled strictly, as it may cause excessive settlement of the embankment.

(4) The width and the slope of the road shoulder both affect settlement of the pavement and the road shoulder. The more uniform the roadbed load is, the more uniform the settlement in the two-dimensional plane. However, for the stability of the roadbed, a suitable shoulder width and shoulder slope need to be adopted.

Acknowledgments

Authors would like to express great gratitude to National Nature Science Foundation of China (Grant Nos. 12172211). The authors also wish to acknowledge National Key Research and Development Project of China (Grant No. 2019YFC1509800).

Author contributions

Chengjia Tang: Conceptualization, Methodology, Formal analysis, Investigation, Visualization, Writing - original draft. **Lei Wang**: Conceptualization, Methodology, Investigation, Supervision, Writing review & editing. **Sidong Shen**: Conceptualization, Methodology, Investigation, Supervision, Writing review & editing. **Minjie Wen**: Investigation, Supervision, Writing - review & editing. **Annan Zhou**: Investigation, Supervision, Writing - review & editing.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Electronic supplementary materials

Sections S1–S5

中文概要

题　目： 非均匀梯形荷载下非饱和土二维平面应变固结特性研究

作　者： 唐程嘉[1]，汪磊[1]，沈思东[1]，闻敏杰[2]，周安楠[3]

机　构： [1]上海工程技术大学，城市轨道交通学院，中国上海，201620；[2]浙江大学，软土与地质环境工程教育部重点实验室，浙江杭州，310058；[3]墨尔本皇家理工学院，土木、环境和化学工程学院，澳大利亚墨尔本，VIC 3001

目　的： 在公路工程建设中，为使公路安全稳定，必须将原地面填筑成梯形路堤，而路堤的大小同时在宽度和高度上都有变化，这就造成了地面上的非均匀荷载。然而，目前关于固结特性的研究并没有考虑外部荷载沿水平方向的变化。在以往的研究中，外部荷载被假定为随时间变化的、理想的均匀荷载。此外，由于气候条件的影响，实际工程中处理的通常是非饱和路基。因此，本文推导了非均匀荷载下的非饱和土的二维平面应变固结的半解析解，我们选择梯形荷载作为非均匀荷载的特例，这也更接近于底层非饱和地基上的堤坝荷载。以此来研究非均匀梯形荷载下的非饱和土的二维平面应变固结特性。

创新点： 1. 基于 Dakshanamurthy 和 Fredlund 二维平面应变固结理论，建立非均匀梯形荷载下非饱和土二维平面应变固结的参考模型；2. 通过拉普拉斯变换与傅里叶展开推导了非均匀荷载下的非饱和土的二维平面应变固结的半解析解

方　法： 1. 建立非均匀梯形荷载下非饱和土二维平面应变固结的参考模型（图 1）；2. 推导了非均匀荷载下的非饱和土的二维平面应变固结的半解析解（公式(6)(7)(9)）；3.绘制超孔隙压力与沉降在非均匀荷载下的变化图像，分析不同计算点、路肩宽度与路堤高度对固结特性的影响。

结　论： 1. 不同计算点的超孔隙压力消散速度不同。路面下的超孔隙压力比路肩处的大很多，消散开始得晚，但消散速率快；2. 路堤高度的增加会增加路面的最终沉降，但对路肩的影响较小；3. 路肩的宽度和坡度都会影响路面和路肩的沉降。路基荷载越接近于均匀荷载，二维平面上的沉降就越均匀。

关键词： 非饱和土；非均匀荷载；二维平面应变固结；半解析解；拉普拉斯变换；傅里叶级数展开